THEORY & OBJECTIVE

STRENGTH OF MATERIAL

By
Team of
Engineers Academy

- Covers ESE, GATE, IAS, PSUs, DRDO, ISRO & other Technical Exams.
- Covers complete syllabus & all topics of ESE & GATE examination.
- Thoroughly revised, fully solved & error free book.
- Concise, concept oriented and topicwise presentation & authentic solutions.

ENGINEERS ACADEMY™
Your GATEway to Professional Excellence
IES • GATE • PSUs • JTO • IAS • NET

CORPORATE OFFICE
100-102, Ram Nagar, Bambala Puliya, Tonk Road, Pratap Nagar, Jaipur-302033
Ph. : 0141-6540910, +91-8094441777
Website : www.engineersacademy.org | **Email:** info@engineersacademy.org

Ajmer | Jaipur | Kota | Jodhpur | Bhilwara | Delhi | Patna | Lucknow | LPU | Ludhiana | Jalandhar | Kanpur

CONTENTS

❑❑❑

1 INTRODUCTION

THEORY

1.1 MATERIAL CLASSIFICATION

According to behaviour on loading, material can be classified as

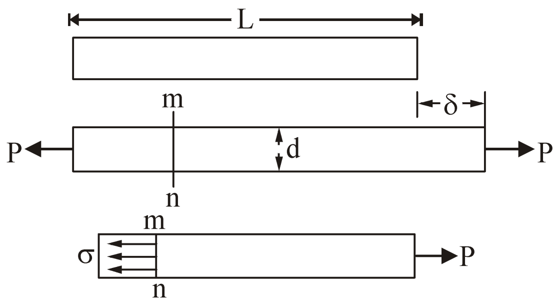
1. **Elastic :** Undergoes deformation when subjected to the external loading and comes back to its original state after removal of load.
2. **Plastic :** Material do not regain its original dimensions and the deformation is permanent.
3. **Rigid :** Does not undergo any deformation when loaded externally.

In statics and dynamics, we deal with forces and motions associated with particles and rigid bodies. In strength of materials, we examine the stresses and strains that occur inside real bodies those deform under loads, here you must understand the difference between rigid body and real body.

1.2 STRESS AND STRAIN

1.2.1 Normal Stress

Consider a prismatic bar loaded by axial forces P at the ends. A prismatic bar is straight structural member having constant cross section throughout its length. The axial force produce a uniform stretching of bar. Here, bar is said to be in tension.



A section taken perpendicular to longitudinal axis of bar is cross-section. Considering free-body diagram, the tensile force P acts on right hand of free body at the other end is force representing the action of removed part of bar upon the part that remains. These forces are continuously distributed over the cross-section. The intensity of force (i.e. force per unit area) is called the stress and is denoted by

Hence under equilibrium, $F = \sigma A$

$$\Rightarrow \quad \sigma = \frac{F}{A}$$

The stress is the force of resistance per unit area offered by a body against the deformation.

When the bar is stretched by force P, as shown in figure, the resulting stresses are tensile stresses and if forces are reversed in direction, causing the bar to be compressed, the stresses are compressive stresses.

As stress acts in direction perpendicular to cut surface, it is referred as normal stress. The normal stresses may be tensile or compressive. The shear stress act parallel to the surface. Conventionally the tensile stresses are taken as positive and compressive stresses are negative.

The unit of stress is N/m^2 also referred as pascal.

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ Kgf/cm}^2 = 0.1 \text{ MPa}$$

$$1 \text{ mPa} = 10^{-3} \text{ N/m}^2$$

It can also be expressed as MPa. i.e., N/mm^2.

1.2.2 Normal Strain

An axially loaded bar undergoes a change in length, becoming longer in tension and shorter when in compression, strain is defined as change in length per unit length.

$$\text{Strain } (\in) = \frac{\delta}{L}$$

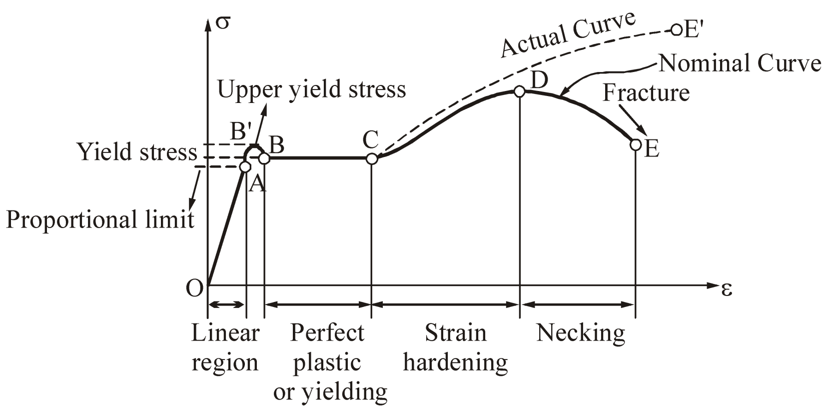
If bar is in tension, the strain is called a tensile strain, representing stretching of material. If the bar is in compression, the strain is compressive strain. The tensile strain is taken as positive and compressive strain is negative. The strain $\in$ is called normal strain because it is associated with normal stresses. As normal strain $\in$ is ratio of two lengths it is a dimensionless quantity i.e. it has no units.

Note : The definition of normal stress and normal strain are based purely on statical and geometrical consideration. It can be used for load of any magnitude and for any material.

1.3 STRESS STRAIN DIAGRAM OF MILD STEEL IN TENSION

The mechanical properties of material are determined by test performed on small specimen of the material. The most common test is tension test, in which tensile loads are applied on cylindrical specimen. The American Society for Testing and Materials (ASTM) standard tension specimen has diameter of 0.5 in and a gauge length of 2.0 in. The machine used in test is Universal Testing Machine (UTM).

In a static test, the load is applied very slowly and in dynamic test, the rate of loading may be very high. Here, we are analyzing properties based on static test.



The typical stress strain diagram of mild steel is shown in figure. Here, the stress is nominal stress or engineering stress and strain is nominal strain or engineering strain.

$$\text{Nominal stress} = \frac{\text{Load}}{\text{Initial cross section area}} = \frac{P}{A_0}$$

$$\text{True stress} = \frac{\text{Load}}{\text{Actual Area}} = \frac{P}{A_a}$$

$$\text{Nominal strain} = \frac{\Delta L}{L_0}$$

$$\text{True strain} = \frac{\Delta L}{L_a}$$

The nominal stress is obtained by dividing the load P by initial cross sectional area A. The true stress is calculated by using the actual area of the bar.

Similarly, for calculation of strain, if initial gauge length is used nominal strain is obtained. If the actual length is used, true strain is obtained.

1. The diagram begin with straight line from O to A. In this region the stress and strain are directly proportional and behaviour of material is linearly elastic.
2. Beyond point A, linear relationship between stress and strain no longer exists. A is called proportional limit.
3. When load is increased beyond A, the slope of curve become smaller and smaller, unit at point B, the curve becomes horizontal.
4. From B to C, considerable elongation occurs with no increase in tensile force. The phenomenon is known as yielding, region BC is called as yield plateau.
5. In region CD, the material begin to strain hardening, the material undergoes change in its atomic and crystalline structure, resulting in increased resistance of material to further deformation.
6. The load reaches its maximum value and corresponding stress is called ultimate stress.
7. The fracture finally occur at point E as shown in figure. Various properties of material can be deduced from stress-strain diagram, stress corresponding to E is called fracture/rupture stress.

1.4 Some Important Properties of Material

1.4.1 Ductility

The ductility of material by which it can be drawn as wire of small cross-section upon tensile forces.

The percent elongation is defined as

$$\text{Percent elongation} = \frac{L_f - L_0}{L_0} \times 100$$

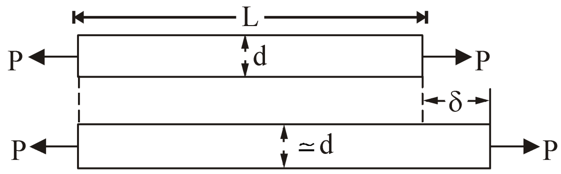
$$\text{Percent change in area} = \frac{A_f - A_0}{A_0} \times 100$$

2 AXIALLY LOADED MEMBERS

THEORY

2.1 DEFLECTION OF BAR

Consider a prismatic bar of length L loaded in tension by axial force P.



$$\sigma = \frac{P}{A}$$

$$E = \frac{\sigma}{\in} = \frac{P}{A}\frac{L}{\delta}$$

$\Rightarrow$ $$\delta = \frac{PL}{AE}$$

The deflection in bar, $$\delta = \frac{PL}{EA}$$

Energy stored in bar = work done, $$U = \frac{1}{2}P\delta$$

$$= \frac{1}{2}\times P\times\frac{PL}{AE} = \frac{P^2L}{2AE}$$

Where, P = Applied load

L = Length

A = Cross-sectional area

E = Modulus of elasticity

The stiffness k of axial loaded bar is defined as force required to produce a unit deflection

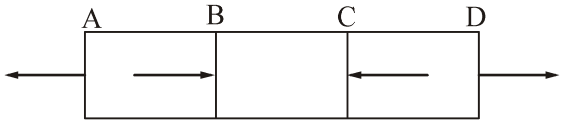
$$k = \frac{EA}{L}$$

The flexibility (f) is defined as deflection due to unit load.

$$f = \frac{L}{EA}$$

2.1.1 Principle of Superposition

According to principle of superposition the resulting strain will be equal to algebraic sum of strain caused by individual forces acting along the length of the member.



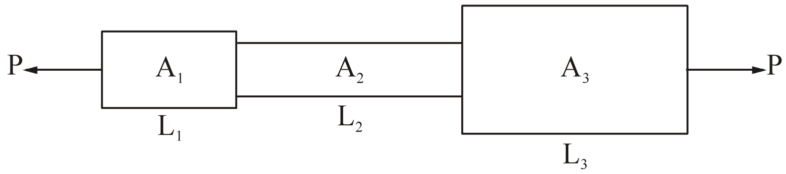
Considering the free body diagram of different section of bar, total deformation can be calculated as

$$\Delta = \Sigma\frac{PL}{AE} = \frac{1}{AE}\left(P_1L_1 + P_2L_2 + ... + P_nL_n\right)$$

Where : P_1, P_2, ... P_n are axial load on individual sections in their FBD.

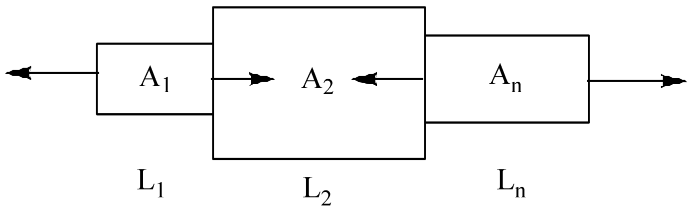
2.1.2 Bar of Varying Cross-section

When a structural member having varying areas of cross-section along its length is subjected to axial force P, the total deformation is equal to sum of deformation of individual section under action of axial force P.



Total elongation is $$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$$

Similarly if bar of varying cross-section is subjected to various forces, at both the ends as well as intermediate points, principle of superposition is applied and total deformation can be computed by drawing the free body diagram of individual section.



$$\Delta = \Sigma\Delta = \frac{1}{E}\left(\frac{P_1L_1}{A_1} + \frac{P_2L_2}{L_2} + ... + \frac{P_nL_n}{A_n}\right)$$

2.1.3 Uniformly Tapering Circular Bars

Let us now consider a uniformly tapering circular bar, subjected to an axial force P, as shown in figure. The bar of length L has a diameter d_1 at one end and d_2 at the other end ($d_2 > d_1$).

Consider a very short section XX of length dx and diameter d_x, situated at a distance x from end A.

Diameter $d_x = d_1 + \frac{d_2 - d_1}{L}x = d_1 + kx$ where $k = \frac{d_2 - d_1}{L}$.

Extension of the short length $= \delta\Delta = \dfrac{P.dx}{\frac{\pi}{4}d_x^2E}$

3

Shear Force and Bending Moment

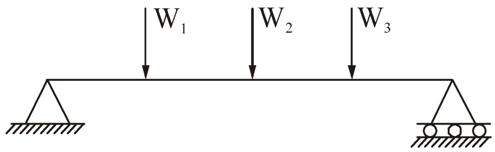
THEORY

3.1 Analysis of Beams

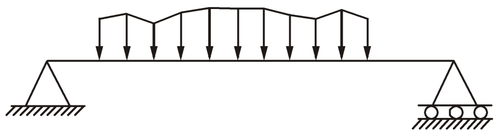
Beam is a structural member that is loaded by system of external loads that acts transverse to its axis. In addition to the point loads or uniformly distributed loads acting transverse to its axis, a beam may also be loaded with force couples acting in a plane passing through axis of the bar.

Types of loads :

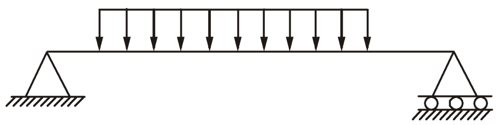
(i) Point Load



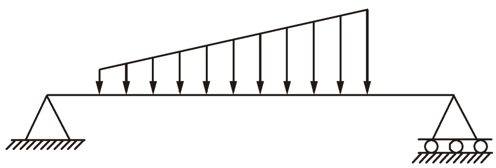
(ii) Distributed Load



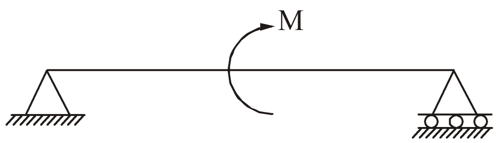
(iii) Uniformly Distributed Load



(iv) Uniformly varying Load



(v) Couple

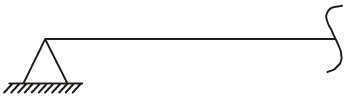


A point load is the one which is considered to act at a point. Distributed loads are those loads which act over some area. Such loads are measured by their intensity which is expressed by force per unit distance along axis of the beam. A uniformly distributed load is one which has uniform intensity.

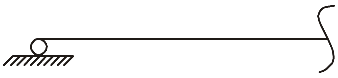
Types of Support :

(A) External Supports : A beam may have following type of external supports.

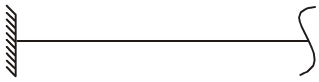
(i) Hinged support



(ii) Roller support



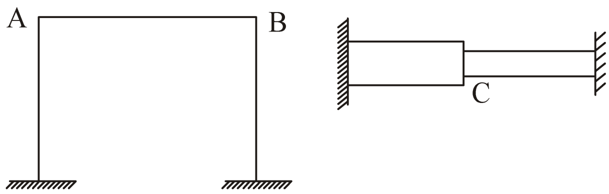
(iii) Fixed support



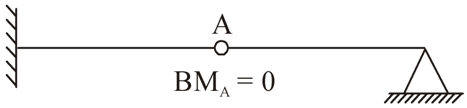
A hinged support is the one at which beam is free to rotate but translational displacement is not possible. In roller support, beam is free to translate along the rolling plane. In fixed support, neither rotation is possible nor translational displacement is possible.

(B) Internal Supports :

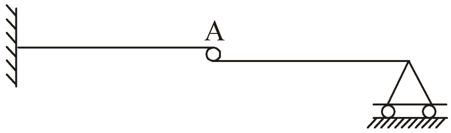
(i) Rigid support



(ii) Internal hinged support



(iii) Internal roller support



$$A.F_A = 0$$
$$B.M_A = 0$$

A rigid support is one which transfer all three forces (S.F , B M, A.F). Internal hinge does not transfer bending moment, So it adds a equation to statical analysis. At internal roller both axial force & Bending moment has to be zero.

Types of beam : Based on support conditions, the beams can be

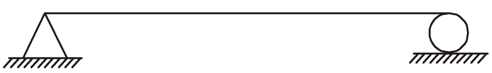
(i) Simply supported beams

(ii) Over hanging beams

(iii) Fixed beam

(iv) Continuous beam.

A simply supported beam is the one in which reaction at both the ends are vertical. Such a beam is free to rotate at the ends, when it bends.

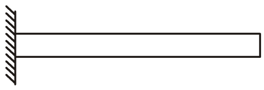


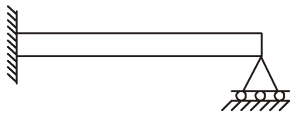
A simply supported beam may have overhang either at one end or at both the ends. Such beam are known as overhang beams.

Single sided overhang

Double sided overhang

A cantilever beam has one end fixed while other end is free having no support. However, if there is roller support at other end it is known as propped cantilever.





Cantilever Propped Cantilever

Propped cantilever is statically indeterminate.

A continuous beam is one which extends over more than one support.

For the analysis of 2-D frames and beams, there are three equilibrium conditions

(a) Summation of horizontal forces $\Sigma F_H = 0$

(b) Summation of vertical forces $\Sigma F_V = 0$

(c) Summation of moment $\Sigma M = 0$

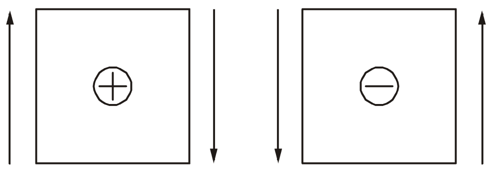
If in a given beam system, number of external reactions are less than or equal to three. Then that beam can be analysed simply by using equilibrium conditions. Such beams and structure are called determinate structure. But if number of unknown reaction are more than three, then additional compatibility equation are needed in order to analyse the structure. Such a structure is said to be indeterminate structure. The degree of indeterminacy is given by number of unknown reaction minus number of available equilibrium condition.

Degree of indeterminacy = Number of unknown reaction – Number of available equilibrium condition. A determinate beam can be stable or unstable.

3.2 Shear Force

It is algebraic sum of all transverse forces either to the left or to the right of section. It always acts along the plane of cross-section.

Sign convention :



If algebraic sum of transverse force to left of section is upward or right of section is downward. Then that shear force will be taken as positive.

3.3 Bending Moment

Bending moment at any section is algebraic sum of all the moment caused by transverse forces and moments either to the left or to the right of that section. Bending moment is always visualizes as particular

60. ***Ans. (a)***

The slope of shear force diagram can be used to find load intensity.

$$\frac{dV}{dx} = -w$$

For point loads and reaction the change in shear force at point of application and support can be used.

Support reaction at A = 14 t

There is no load between A and D as SFD does not change. At D there is sudden change in value of SF from 14 t to – 4t So there is point load of 18 t at point D.

Between D and B the shape of SFD is linearly decreasing so there is uniformly distributed load between B and D. The intensity of UDL is

$$w = \frac{16-4}{8} = \frac{12}{8} = 1.5t/m$$

At B there is sudden change in the SF so there is reaction at B, R_B = 16 + 9 = 25 t

Again between B and C the SFD decreases linearly from 9 t to 3t. So load intensity

$$w = \frac{9-3}{4} = 1.5t/m$$

Finally the SFD decreases to zero suddenly at C. So a point load of 3t is applied at C.

61. ***Ans. (c)***

Bending moment M = $\int V dx$

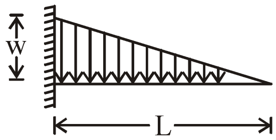
$$V = \int V dx$$

If loading is of nth degree then BM is of (n + 2) degree.

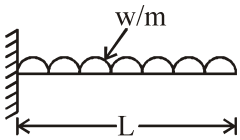
62. ***Ans. (c)***

The vertical component of reaction at A and B will be in the upward direction and horizontal component should be equal to each other.

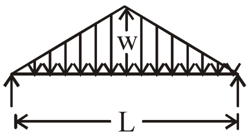
63. ***Ans. (a)***

- 

$$M_{max} = \left(\frac{wL}{2}\right) \times \frac{L}{3} = \frac{wL^2}{6}$$

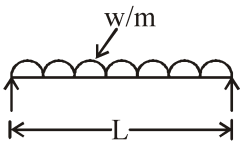
- 

$$M_{max} = wL \times \frac{L}{2} = \frac{wL^2}{2}$$

- 

$$M_{max} = R_A \times \frac{L}{2} - \frac{w(L/2)^2}{6}$$

$$= \frac{wL}{4} \times \frac{L}{2} - \frac{wL^2}{24} = \frac{wL^2}{12}$$

- 

$$M_{max} = R_A \times \frac{L}{2} - \frac{w(L/2)^2}{2}$$

$$= \frac{wL}{2} \cdot \frac{L}{2} - \frac{wL^2}{8} = \frac{wL^2}{8}$$

64. ***Ans. (a)***

$$\frac{dM}{dx} = V$$

$$\therefore \quad M_2 - M_1 = \int V dx$$

= Area under shear curve

65. ***Ans. (b)***

Rate of change of bending moment

$$\frac{dM}{dx} = V$$

66. ***Ans. (b)***

Stresses in Beam

THEORY

4.1 Theory of Pure Bending

Assumption :

(i) Material is isotropic, homogenous and loading is within elastic limit i.e. Hooke's is valid.

(ii) Plane section remains plane even after bending.

(iii) Structure is symmetric and loading passes through centroid of structure.

Equation of pure bending, $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ (Flexure formula)

Where, M = Bending moment

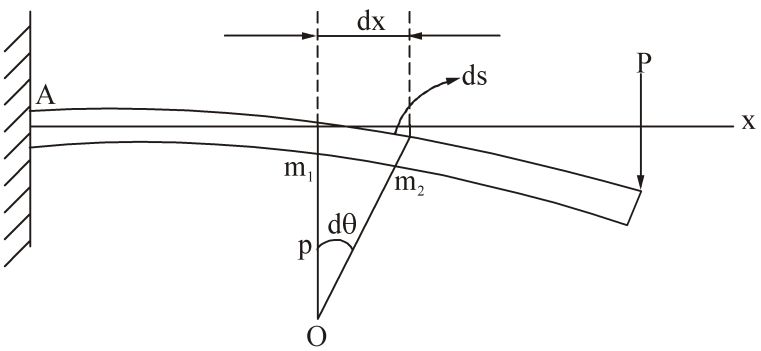
I = Moment of Inertia about bending axis

σ = Shear stress at a distance y from bending axis

E = Modulus of elasticity

R = Radius of curvature

Derivation of this formula is as :



Where, O = centre of curvature

p = radius of curvature

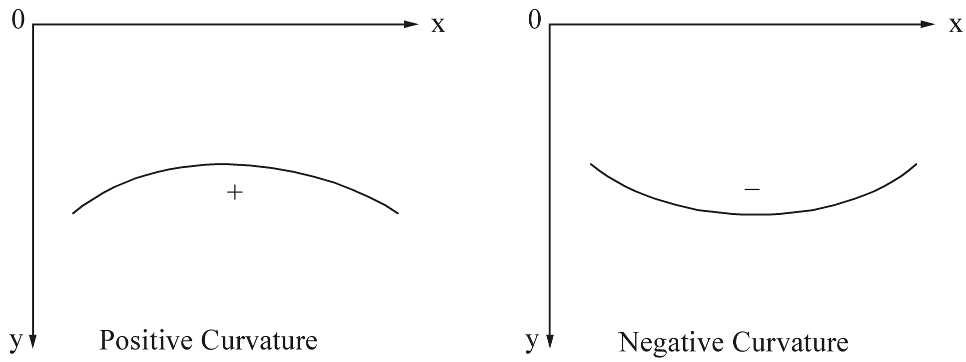
$$ds = pd\theta$$

$\because$ $$ds \approx dx$$

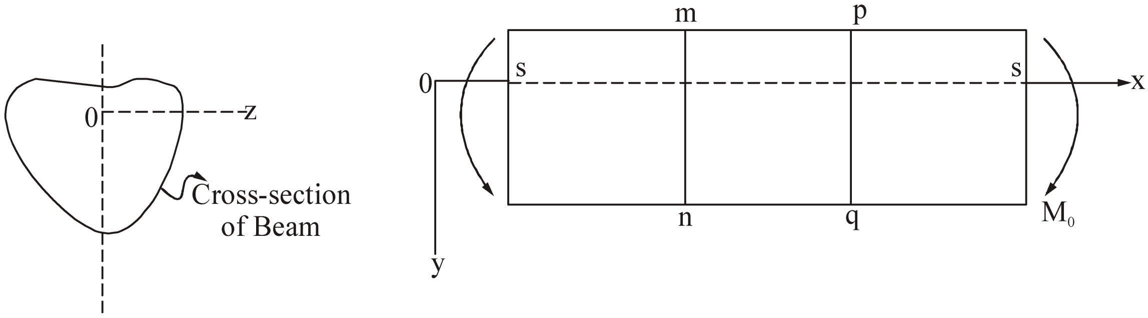
$\Rightarrow$ $$dx = pd\theta$$

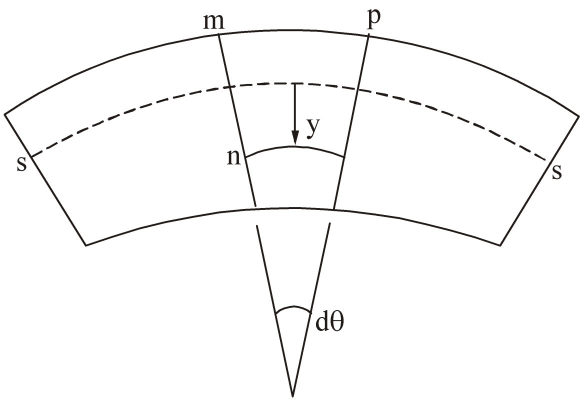
$\Rightarrow$ $$k = \frac{1}{p} = \frac{d\theta}{dx}$$

The sign convention for curvature



Normal strain in beam : For simple bending, the beam is symmetric about xy – plane





SS is neutral surface and its intersection with any cross-sectional plane is called neutral axis of the cross-section.

Normal strain,

$$\epsilon_x = \frac{(p - y)d\theta - pd\theta}{pd\theta} = -\frac{y}{p} = -ky$$

Transverse strain

$$\epsilon_z = -\mu\epsilon_x = \mu ky$$

Normal stresses in beams : From Hooke's law

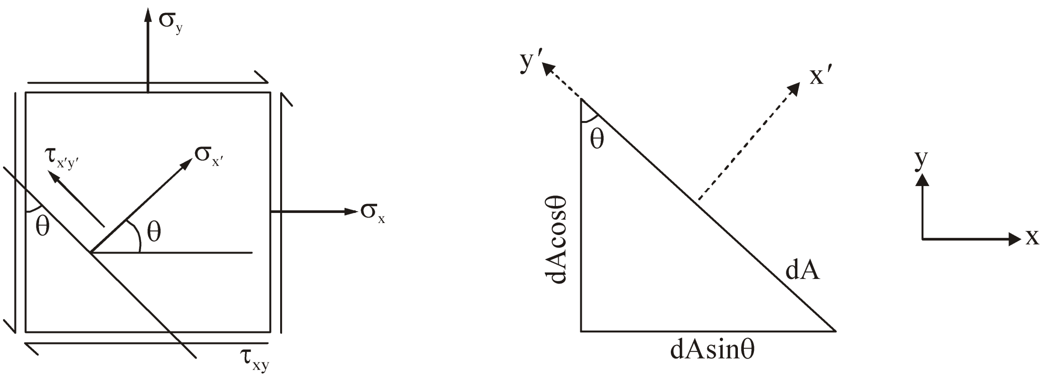
$$\sigma_x = E\epsilon_x = -Eky$$

5

ANALYSIS OF STRESS AND STRAIN

THEORY

5.1 PRINCIPAL STRESSES



On plane stress element when normal stress and shear stress acting on it. For stress transformation,

From the equations of static equilibrium :

$\Sigma F_{x'} = 0$ $\qquad \sigma_{x'} \cdot dA - (\sigma_x\, dA \cos\theta)\cos\theta - (\tau_{xy}\, dA \cos\theta)\sin\theta$
$- (\sigma_y\, dA \sin\theta)\sin\theta - (\tau_{xy}\, dA \sin\theta)\cos\theta = 0$ ...(i)

$\Sigma F_{y'} = 0$ $\qquad \tau_{x'y'} \cdot dA + (\sigma_x\, dA \cos\theta)\sin\theta - (\tau_{xy}\, dA \cos\theta)\cos\theta$
$- (\sigma_y\, dA \sin\theta)\cos\theta + (\tau_{xy}\, dA \sin\theta)\sin\theta = 0$ ...(ii)

On solving equation (i) and (ii) we get the following stress transformation equations :

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad \text{...(A)}$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad \text{...(B)}$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2(\theta + 90°) + \tau_{xy}\sin 2(\theta + 90°) \qquad \text{...(C)}$$

From equation (A) & (C) we get,

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

i.e, sum of the normal stresses acting on perpendicular face of a plane stress element is constant. Hence, it is independent of angle θ.

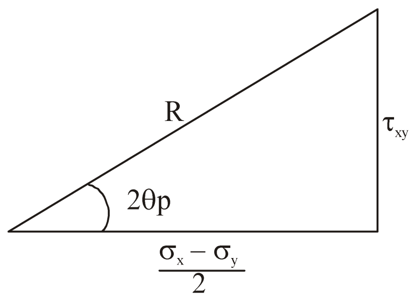
By taking derivative of $\sigma_{x'}$ w.r.t. θ and setting it equal to zero, we obtain an equation that can be solved for value of θ at which $\sigma_{x'}$ is maximum or minimum.

$$\tan 2\theta_P = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

It gives two values of θ_P differ by 90°, so we conclude that the principal stresses occur on two mutually perpendicular planes. The value of principal stresses can be obtained by substituting each of two values of θ_P into stress transformation equation. By this procedure we also know which of two principal stresses is associated with each of two angles θ_P.

$$\cos 2\theta_P = \frac{\sigma_x - \sigma_y}{2R}$$

and $$\sin 2\theta_P = \frac{\tau_{xy}}{R}$$



Hence $$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_2 can be obtained from $\sigma_x + \sigma_y = \sigma_1 + \sigma_2$

$\Rightarrow$ $$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Value of θ_P which satisfy both equations, gives maximum principal stress and other angle θ_{P2} can be taken as 90° larger or smaller than θ_{P1}. It is important to note that shear stress is zero on principal planes. To obtain maximum shear stress and plane on which it act set $\frac{d\tau_{x'y'}}{d\theta} = 0$

$\Rightarrow$ $$\tan 2\theta_S = \frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

i.e., $$\theta_S = \theta_P \pm 45°$$

It implies that plane of maximum shear stress occur at 45° to the principal plane.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$