

THEORY & OBJECTIVE

# THEORY OF STRUCTURES

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# CONTENTS

S.No.	Topic	Page No.
1.	Determinacy Indeterminacy .....	1 – 18
	Objective Sheet .....	19 – 32
2.	Force Method of Analysis .....	33 – 40
	Objective Sheet .....	41 – 47
3.	Displacement Method of Analysis .....	48 – 55
	Objective Sheet .....	56 – 62
4.	Moment Distribution Method .....	63 – 72
	Objective Sheet .....	73 – 90
5.	Truss .....	91 – 102
	Objective Sheet .....	103 – 111
6.	Influence Line Diagram .....	112 – 126
	Objective Sheet .....	127 – 139
7.	Matrix Methods of Analysis .....	140 – 153
	Objective Sheet .....	154 – 159
8.	Arches .....	160 – 173
	Objective Sheet .....	174 – 178



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# DETERMINACY INDETERMINACY

## THEORY

### 1.1 EQUATION OF STATIC EQUILIBRIUM

In a 2-D structure or plane structure (in which all members and forces are in one plane only), the equation of equilibrium are

$$\left. \begin{array}{l} \Sigma F_x = 0 \\ \Sigma F_y = 0 \\ \Sigma M_z = 0 \end{array} \right\} 3 \text{ number}$$

In a 3-D structure or space structure (in which members and forces are in 3-D), the equations of equilibrium are

$$\left. \begin{array}{ll} \Sigma F_x = 0 & \Sigma M_x = 0 \\ \Sigma F_y = 0 & \Sigma M_y = 0 \\ \Sigma F_z = 0 & \Sigma M_z = 0 \end{array} \right\} 6 \text{ number}$$

If member forces cannot be found by equations of static equilibrium alone, the structure is called statically indeterminate.

In this case additional equation needed are obtained by relating the applied loads and reactions to the displacement or slopes known at different points on the structure. These equations are called **compatibility equations**.

### 1.2 DEGREE OF STATIC INDETERMINACY ( $D_S$ )

$$D_S = \left( \begin{array}{l} \text{No. of unknown forces in members} \\ + \text{unknown support reactions} \end{array} \right) - \left( \begin{array}{l} \text{Available equations of} \\ \text{static equilibrium} \end{array} \right)$$

and  $D_S = D_{Si} + D_{Se}$

Where,  $D_S$  = Total indeterminacy

$D_{Si}$  = Degree of internal static indeterminacy

$D_{Se}$  = Degree of external static indeterminacy

### 1.3 SUPPORT REACTIONS

Restraining of deformation at support gives rise to support reactions.

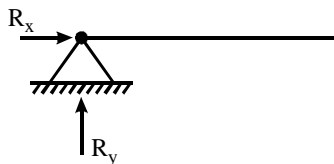
1.3.1 Plane Structure

(1) Fixed support



Fixed support restrains  $\Delta_x$ ,  $\Delta_y$  and  $\theta_{xy}$ . Hence support reactions are  $R_x$ ,  $R_y$  and  $M_z$  (3 nos.)

(2) Pin support or hinged support



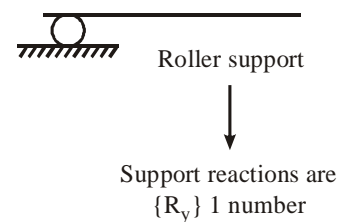
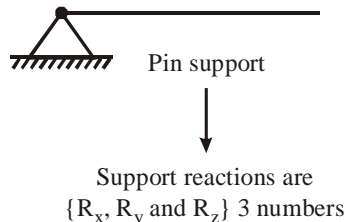
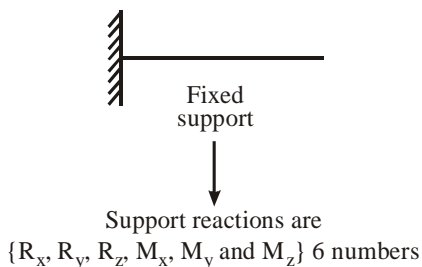
Pin support restrains  $\Delta_x$  and  $\Delta_y$ . Hence support reactions are  $R_x$  and  $R_y$  (2 nos.)

(3) Roller support



Roller support restrains  $\Delta_y$ . Hence support reaction is  $R_y$ .

1.3.2 Space Structure



1.4 EXTERNAL INDETERMINACY ( $D_{Se}$ )

$$D_{Se} = \left( \begin{array}{c} \text{Total no. of support reactions} \\ \text{in the structure} \end{array} \right) - \left( \begin{array}{c} \text{Available equations of} \\ \text{static equilibrium} \end{array} \right)$$

For plane structure

$$D_{Se} = R_e - 3$$

For space structure

$$D_{Se} = R_e - 6$$

1.5 INTERNAL INDETERMINACY ( $D_{Si}$ )

$$D_{Si} = D_S - D_{Se}$$

= Total indeterminacy – External indeterminacy

1.6 DEGREE OF STATIC INDETERMINACY FOR FRAMES

Frames are rigid jointed structures. All the joints are made rigid by providing extra restraint  $R'$ . The structure is then cut to make it, *Open Tree* like determinate structure.

For plane frames  $D_S = 3C - R'$

For space frames  $D_S = 6C - R'$

Where,  $C$  = Number of cuts to make structure determinate

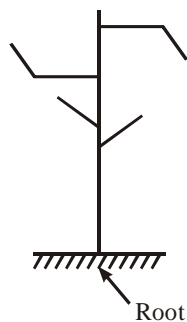
$R'$  = Number of restraints applied to make all joints rigid.

1.6.1 Open Tree Like Structure

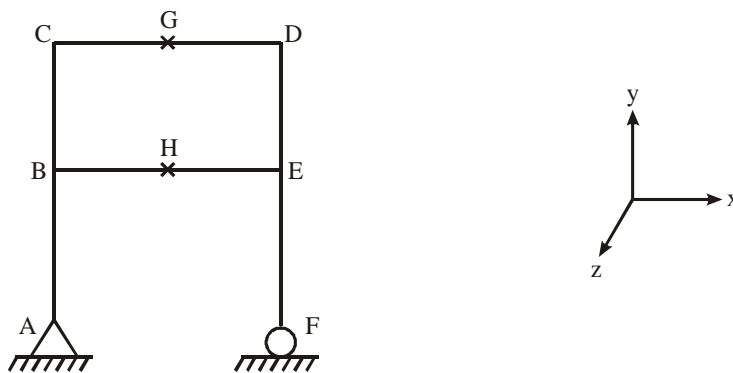
The structure is cut in such a way that each individual cut part looks like a tree as shown below.

Note that :

1. Tree should have only one root.
2. Tree cannot have a closed looped branch.



Ex.:



Since it is a plane frame.

$$D_S = 3 \times 2 - (3) = 3$$

Number of restraint that is required to be applied to make the structure rigid is

1. One number at support A i.e. moment  $M_{Az}$ .
2. Two numbers at support F i.e.  $R_{Fx}$  and  $M_{Fz}$ .

Thus, total number of added restraints =  $R' = 1 + 2 = 3$

This  $R' = 3$  corresponds to 3 known reaction conditions i.e.  $M_{Az} = 0$ ,  $M_{Fz} = 0$  and  $R_{Fx} = 0$ .

When the structure is cut it gets divided into two parts.



# FORCE METHOD OF ANALYSIS

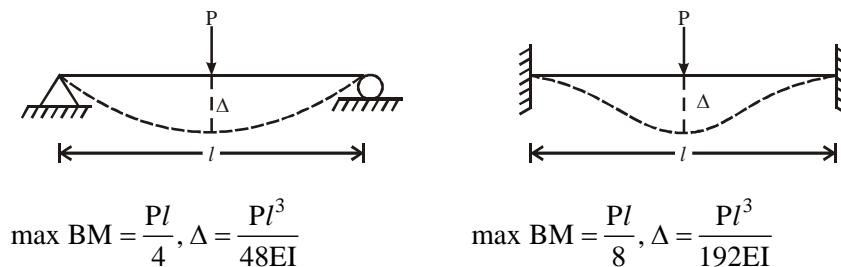
## THEORY

### 2.1 INTRODUCTION

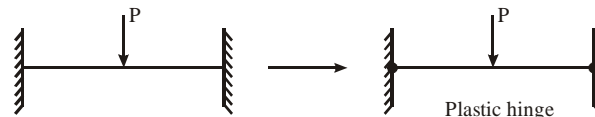
Purpose of choosing statically indeterminate structure as compared to statically determinate ones are as follows.

(1) For a given loading the maximum stress and deflection of an indeterminate structures are generally smaller than that in statically determinate structure.

Ex.:



(2) Indeterminate structures have a tendency to redistribute its load to its redundant supports in case where faulty design or overloading occurs.



As load P is increased, plastic hinge will form at supports first and hence it will be treated as simply supported structures. Further load can also be resisted. However, in case of simple supports hinge will form at centre hence collapse occurs earlier.

Although there will be cost saving in material due to lesser stress in member, the cost of construction of **supports** and **joint** may some time offset the saving in material.

Differential settlement of supports, temperature variation, change in length due to fabrication errors in indeterminate structures will introduce internal stresses in structure.

**Note :** In statically determinate structures internal stresses will not be introduced because of these factors.

In any statically indeterminate structures, it is necessary to satisfy

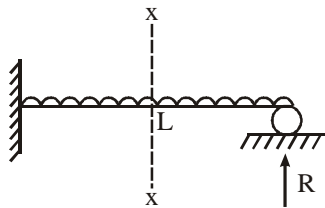
1. Equilibrium equations
2. Compatibility equations
3. Force displacement requirement

There are two different methods to satisfy above requirements

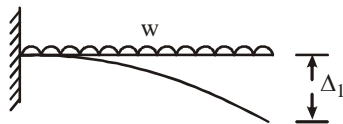
- (1) Force method
- (2) Displacement method

Force Method	Displacement Method
<ol style="list-style-type: none"> <li>1. Force method is also called as Method of consistent deformation or Flexibility method</li> <li>2. Unknowns in this method are Forces (Reactions i.e. BM, SF)</li> <li>3. Force-displacement equations are written and solution for unknown forces is obtained from compatibility equations.</li> <li>4. Once unknown forces are known, the reactions forces like BM, SF etc. are found using equilibrium equation.</li> </ol>	<ol style="list-style-type: none"> <li>1. Also called : compatibility method</li> <li>2. Unknowns in this method are Displacement (<math>\Delta</math>, <math>\theta</math>).</li> <li>3. Force-displacement equations are written and solution for unknown displacement is obtained from equilibrium equations.</li> <li>4. Once unknown displacements are known, internal forces are found using compatibility and load-displacement equation.</li> </ol>

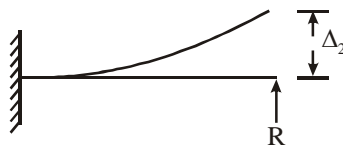
Example



$R =$  redundant



$$\Delta_1 = \frac{wL^4}{8EI}$$



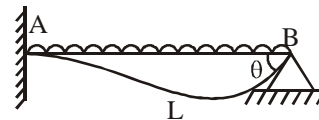
Load displacement relationship,

$$\Delta_2 = \frac{RL^3}{3EI}$$

Compatibility equations,  $\frac{wL^4}{8EI} = \frac{RL^3}{3EI}$

Equilibrium equation,

$$BM_x = Rx - \frac{Wx^2}{2}$$



$\theta =$  unknown displacement

Load displacement relation

$$M_{AB} = M_{FAB} + \frac{2EI}{L}(-\theta_B)$$

$$\text{and } M_{BA} = M_{FBA} + \frac{2EI}{L}(-2\theta_B)$$

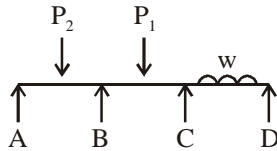
Equilibrium equation,  $M_{BA} = 0$

$$\Rightarrow \frac{4EI\theta_B}{L} = M_{FBA} = \frac{-wL^2}{12}$$

$$\Rightarrow \theta_B = \frac{wL^3}{48EI}$$

Using  $\theta_B$ ,  $M_{AB}$  can be found using load displacement relationship.

5. Force method of analysis of indeterminate structure is suitable when degree of static indeterminacy is less than degree of kinematic indeterminacy.

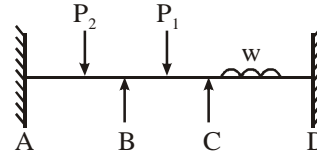


No. of unknown reactions = 2  
 No. of unknown joint displacements =  $4(\theta_A, \theta_B, \theta_C, \theta_D)$

Hence, force method is suitable.

6. Examples of force methods
- (i) Castiglano's theorem-II (method of least work).
  - (ii) Strain energy method.
  - (iii) Virtual work method/Unit load method.
  - (iv) Clapeyron's three moment method. (used in continuous beam analysis).
  - (v) Column analogy method (used in rigid frames with fixed supports).
  - (vi) Flexibility matrix method.

5. Displacement method is suitable when degree of kinematic indeterminacy is less than degree of static indeterminacy.



Unknown reaction = 4  
 Unknown displacement  $(\theta_B, \theta_C) = 2$

Displacement method is suitable.

6. Examples of displacements methods
- (i) Slope deflection method.
  - (ii) Moment distribution method.
  - (iii) Stiffness matrix method.
  - (iv) Kani's method.

## 2.2 FUNDAMENTAL ASSUMPTIONS IN STRUCTURAL ANALYSIS

### 2.2.1 Principal of Superposition

The total displacement or internal reaction (stress) at a point in a structure subjected to several external loading can be determined by adding together the displacements or internal reaction (stress) caused by each of the external loads acting separately.

Two requirements for principal of superposition are

1. Linear elastic response i.e. Hooke's law valid.  
 Stress  $\propto$  Strain
2. Small displacement theory applies i.e., geometry of the structure must not undergo significant change when loads are applied.

The above requirements are also the requirements for **Linear 1<sup>st</sup> Order Analysis**. Hence in our course we follow linear 1<sup>st</sup> order analysis.



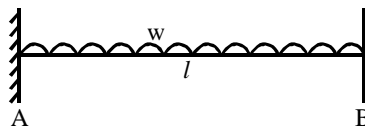
# DISPLACEMENT METHOD OF ANALYSIS

## THEORY

### 3.1 SLOPE DEFLECTION METHOD

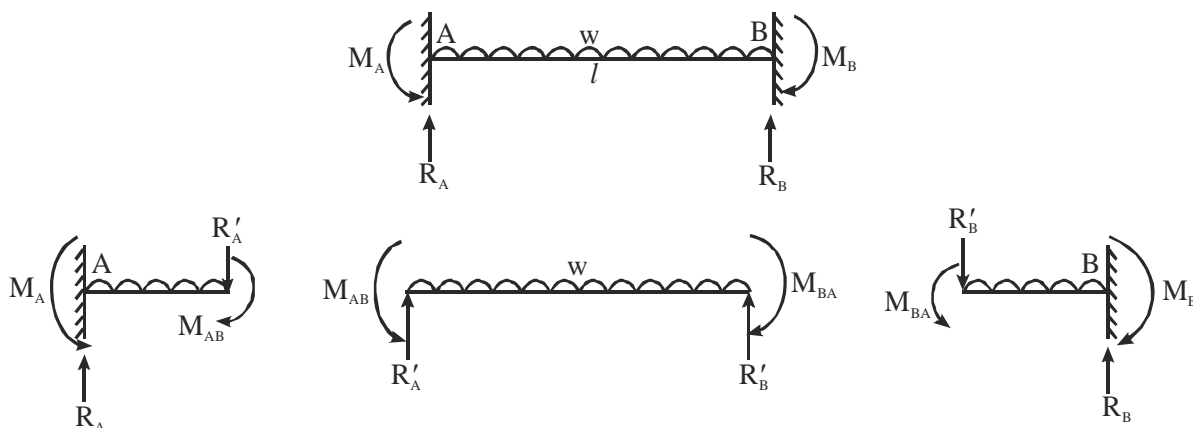
- Slope deflection method is useful to analyze indeterminate structures like continuous beams and plane frames.
- The unknowns in this method are degree of freedom i.e. slope ( $\theta$ ) and deflection ( $\Delta$ ).
- Together these slopes and deflections are known as displacements. Thus slope-deflection method is a displacement method.
- Other examples are moment distribution method, Kani's method and stiffness matrix method are based.
- Analysis of a beam and frame means determination of bending moment and shear force throughout the length of the member i.e. determination of BMD and SFD for the member. BMD and SFD for a member of structure can be drawn if we know the internal forces of a member & end moments.

**Ex.:** If we have a fixed beam as shown below



The BMD and SFD can be found if we know the internal forces and end moments of member i.e.  $M_{AB}$  and

$M_{BA}$



From figure (1) for calculation of reaction  $R_A$  and  $R_B$

First Applying  $\Sigma M_B = 0$

$$\Rightarrow R_A \times l - M_A + M_B - w \times l \times \frac{l}{2} = 0$$

$$\Rightarrow R_A = \frac{wl}{2} + \frac{M_A - M_B}{l}$$

Similarly  $\Sigma M_A = 0$

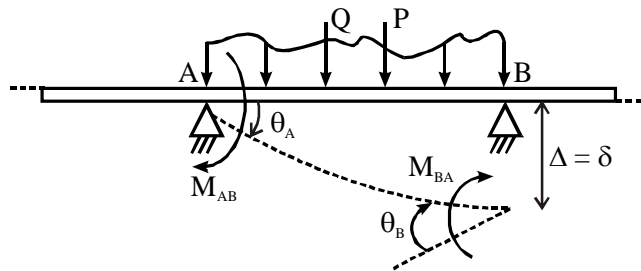
$$\Rightarrow R_B \times l - M_B + M_A - wl \times \frac{l}{2} = 0$$

$$R_B = \frac{wl}{2} + \frac{M_B - M_A}{l}$$

Thus, in slope deflection method we establish a relationship between the degree of freedom ( $\theta$ ,  $\Delta$ ) and the member end moments. This relationship is called slope deflection relationship. Finally by using equilibrium equation we find the slope deflection relationship to obtain the member moments.

To find out slope-deflection relationship, method of superposition is used.

**Ex.:** (Continuous Beam)



Where,  $\Delta$  = deflection of joint B with respect of joint A

$\theta_A$  and  $\theta_B$  = rotation of A and B respectively

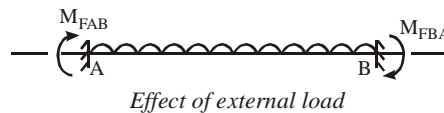
Internal member end moment at A =  $M_{AB}$

Internal member end moment at B =  $M_{BA}$

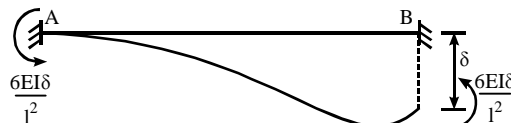
To find out the effect of external loading, rotation  $\theta_A$ ,  $\theta_B$  and displacement 'δ' on internal moments, we follow method of superposition.

### 3.1.1 Steps to Determine Slope Deflection Relationship

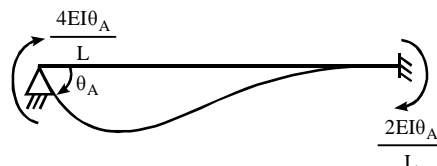
- (i) Consider all the joints to be fixed. The member end moment due to external loads are  $M_{FAB}$  and  $M_{FBA}$  as shown in figure below



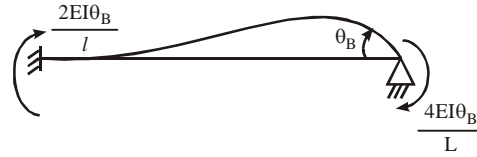
- (ii) Allow the support B to settle with respect to A. The member end moments generated are as show below



- (iii) Allow end A to rotate. The member end moments are as shown below



(iv) Allow end B to rotate. The member end moments are as shown below



(v) Axial deformation are neglected.

(vi) Shear deformation are neglected.

(a) **Sign convention :** (For calculation of member end moment).

- Clockwise moment is taken as positive.
- Clockwise rotation is taken as positive.
- If 'δ' gives clockwise rotation, it is taken as positive. For example in the above discussion if end support B settles by amount 'δ' with respect to A, it is taken as positive.

Thus, from the principal of superposition, we have

$$M_{AB} = M_{FAB} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\delta}{l^2}$$

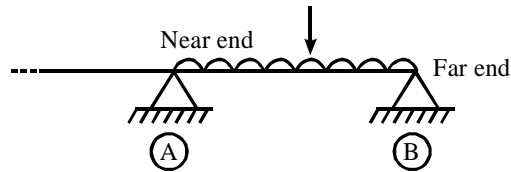
$$M_{BA} = M_{FBA} + \frac{2EI\theta_A}{l} + \frac{4EI\theta_B}{l} - \frac{6EI\delta}{l^2}$$

$$M_{AB} = M_{FAB} + \frac{2EI}{l} \left( 2\theta_A + \theta_B - \frac{3\delta}{l} \right)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l} \left( 2\theta_B + \theta_A - \frac{3\delta}{l} \right)$$

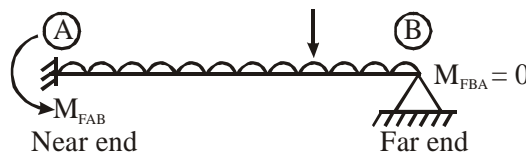
This equation A is called slope deflection equation.

(b) **Slope deflection equation when one end is pin supported**



Steps in the determination of slope-deflection equations are as given below.

(i) Consider the continuous end to be fixed. Member end moment due to external load is  $M_{FAB}$



(ii) Allow the end B to settle by an amount 'δ' with respect to end A. The member end moment is

$$\frac{3EI\delta}{l^2} \text{ as shown below.}$$



# MOMENT DISTRIBUTION METHOD

## THEORY

### 4.1 MOMENT DISTRIBUTION METHOD

#### 4.1.1 Basic Concept

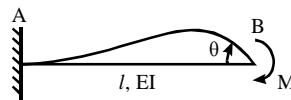
In this method the analysis is done by assuming each joint in the structure to be fixed. Then by unlocking and locking each joint in succession. The internal moments at the joints are distributed and balanced until the joints have rotated to their final or nearly final positions. The following examples have been given to illustrate the basic concept.

#### 4.1.2 Sign Convention

Clockwise moment is taken as positive and anticlockwise moment is taken as negative i.e. the sign convention is same as that in slope deflection method for member end moment calculation.

#### 4.1.3 Member Stiffness Factor

(i)

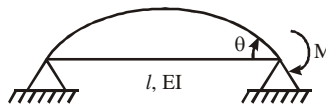


$$\theta = \frac{ML}{4EI}$$

Stiffness,

$$K = \frac{M}{\theta} = \frac{4EI}{L}$$

(ii)



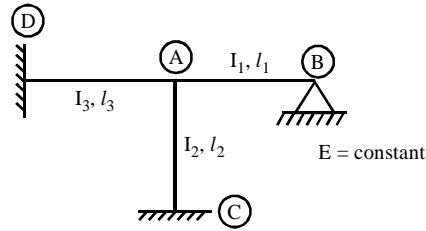
$$\theta = \frac{ML}{3EI}$$

Stiffness,

$$K = \frac{M}{\theta} = \frac{3EI}{L}$$

#### 4.1.4 Joint Stiffness Factor

As joint 'A' is rigidly connected, all members at joint 'A' will rotate by the same amount  $\theta_A$ . If a moment 'M' is applied at joint A, the moment gets distributed in all the connected members. As all connected members rotate by same amount  $\theta_A$  At A. We have



$$M = \frac{4EI_3}{l_3} \theta_A + \frac{4EI_2}{l_2} \theta_A + \frac{3EI_1}{l_1} \theta_A$$

$$\frac{M}{\theta_A} = K_1 + K_2 + K_3 = K$$

Where,  $K$  = Joint stiffness factor

#### 4.1.5 Distribution Factor (DF)

If a moment 'M' is applied at joint 'A' due to which the joint rotates by  $\theta_A$  then

$$\text{Moment distributed in AD} = \frac{4EI_3}{l_3} \theta_A = K_{AD} \theta_A = M_{DAD}$$

$$\text{Moment distributed in AC} = \frac{4EI_2}{l_2} \theta_A = K_{AC} \theta_A = M_{DAC}$$

$$\text{Moment distributed in AB} = \frac{3EI_1}{l_1} \theta_A = K_{AB} \theta_A = M_{DAB}$$

$$\therefore M_{DAD} : M_{DAC} : M_{DAB} = \frac{I_3}{l_3} : \frac{I_2}{l_2} : \frac{3I_1}{4l_1}$$

Where,  $\frac{I_3}{l_3}$  = Relative stiffness of AD

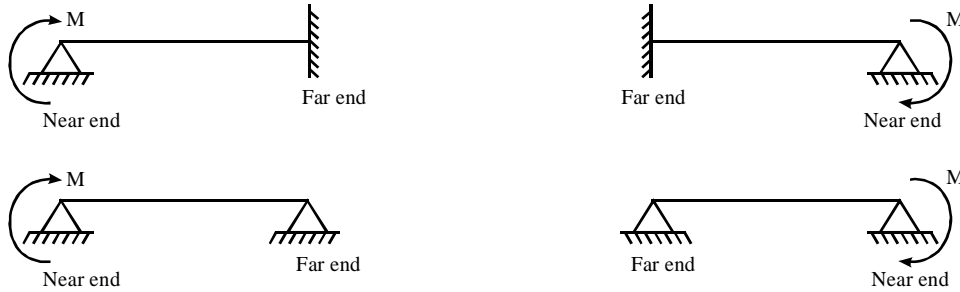
$\frac{I_2}{l_2}$  = Relative stiffness of AC

$\frac{3I_1}{4l_1}$  = Relative stiffness of AB

Hence, Relative Stiffness when for end is fixed =  $\frac{I}{l}$

and Relative Stiffness when for end hinged =  $\frac{3I}{4l}$

**Note :** Meaning of far end



$$\text{Distribution factor for AD} = \frac{M_{DAD}}{M_{DAD} + M_{DAB} + M_{DAC}} = \frac{M_{AD}}{M} = \frac{K_{AD}}{K}$$

$$\text{Distribution factor for AC} = \frac{M_{DAC}}{M} = \frac{K_{AC}}{K}$$

$$\text{Distribution factor for AB} = \frac{M_{DAB}}{M} = \frac{K_{AB}}{K}$$

Hence, **Distribution factor of member** =  $\frac{\text{Stiffness of member}}{\text{Total stiffness of all members at the joint}}$

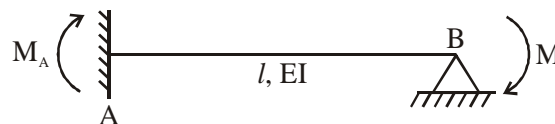
$$\therefore DF_{AD} = \frac{\frac{4EI_3}{l_3}}{\frac{3EI_1}{l_1} + \frac{4EI_2}{l_2} + \frac{4EI_3}{l_3}}$$

Also, **Distribution factor of member** =  $\frac{\text{Relative stiffness of member}}{\text{Total relative stiffness of all members at the joint}}$

$$\therefore DF_{AD} = \frac{\frac{I_3}{l_3}}{\frac{3}{4} \frac{I_1}{l_1} + \frac{I_2}{l_2} + \frac{I_3}{l_3}}$$

### 4.1.6 Carry Over Factor (C.O.F.)

(i) When far end is fixed



Where,  $M$  = Applied moment

$$M_A = \frac{M}{2} = \text{Carried over moment}$$