THEORY & OBJECTIVE

SIGNALS & SYSTEM

By
Team of
Engineers Academy

- State Engineering Services Examinations.
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1

SIGNAL AND ITS PROPERTIES

THEORY

1.1 SIGNALS

Any set of data that contains some information and varies with respect to some independent variables, is called signal.

It may vary with respect to one variable or more than one variables. If it is varies with respect to one variable it is called one dimensional signal and generally this independent variable is time (t). On the other hand if it varies with respect to more than one variable it is called multi-dimensional signal.

Note : In our syllabus we discuss only one-variable signal and this variable is time.

1.2 CHARACTERISTIC OF A SIGNAL

There are following parameters are required to express a signal :

(i) Amplitude :

It signify the strength of the signal, and its unit depends on the type of signal.

(ii) Frequency :

It represents the rate of oscillation of the signal. Its unit is either radian per second or hertz.

(iii) Initial phase :

Initial phase represents whether the given signal is delayed version or advanced version of a standard signal. its unit is radian or degree.

Consider a signal $x(t) = A \cos(\omega_0 t + \phi)$ here

A : Amplitude of signal

ω_0 : Frequency of signal

ϕ : Initial phase of signal

DO YOU KNOW ?
Why always we discuss about sinusoidal signals only ?
As we have seen that by using fourier series and transform any signal can be represent as the sum of sinusoidal signals it means basic building block of all the signals are the sinusoidal signals only.

1.3 CLASSIFICATION OF SIGNALS

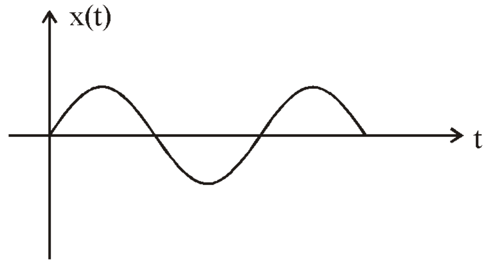
Signals can be classify into following categories :

(i) Continuous time and discrete time signals
(ii) Analog and digital signals
(iii) Energy and power signals
(iv) Periodic and Non-periodic signals
(v) Deterministics and Random signals
(vi) Even and odd signals
(vii) Causal and Non causal signal
(viii) Right sided and left sided signals.

1.3.1 Continuous Time and Discrete Time Signals :

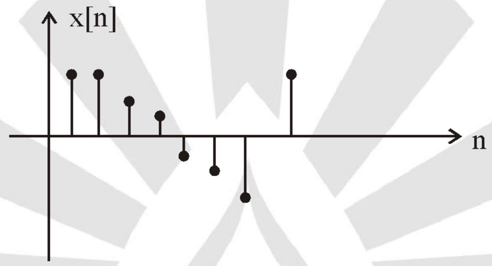
(A) Continuous Time Signals :

A continuous signal is one which is defined for all values of time in its domain :



(B) Discrete Time Signal :

Discrete time signals are those signals which are defined only at discrete value of time:



(C) Conversion of continuous signal into Discrete signal :

A discrete signal is a sampled version of a continuous signal

Consider a continuous signal.

$$x(t) = A\cos(\omega_0 t)$$

Let x(t) is sampled by a sampler having time period is T_s,

Than, $x[n] = A\cos(\omega_0 n T_s)$

$$x[n] = A\cos(\Omega_0 n)$$

Where, $\Omega_0 = \omega_0 T_s$

Ω_0 is called discrete freqency in radion per second and ω_0 is called continuous frequency.

Example: Find the sampled version of x(t) = 4 cos (4t) and hence determine the x[9], if the sampling period is $\frac{\pi}{16}$ seconds.

Solutions: $x(t) = 4\cos(4t)$

So, $x[n] = 4\cos\left(4n\frac{\pi}{16}\right)$

$$x[n] = 4\cos\left(\frac{\pi}{9}n\right)$$

Therefore, $x[9] = 4\cos\left(\frac{9\pi}{4}\right)$

$$= 4\cos\left(2\pi + \frac{\pi}{4}\right) = 4\cos\left(\frac{\pi}{4}\right) = \frac{4}{\sqrt{2}}$$

1.3.2 Analog and Digital Signals :

(A) Analog Signals :

If a signal can take all possible values over a finite range of amplitude axis, it is called analog signal.

(B) Digital Signals :

If a signal can take only some finite set of value over a finite range of amplitude axis, it is called digital signal.

> **DO YOU KNOW ?**
> Binary is a special case of digital signal in which signal can take only two values

(C) Difference between analog and continuous signals :

An analog signal must be continuous over time axis as well as amplitude axis where as a continuous signal is continuous over a only time axis. It may or may not be continuous over amplitude axis.

(D) Difference between discrete and digital signals :

If only time axis is discretize, it is called discrete signal where as if both the axis is discretize it is called digital.

Note: *ADC (Analog to digital converter) is a device used to convert an analog signal into digital signal.*

1.3.3 Energy and Power Signals :

A signal is said to be an energy signal if energy of the signal is finite and power of the signal is zero. i.e. for an energy signal

$$E_g = \text{finite and } P_g = 0$$

(A) Power signal :

A signal is said to be power signal if its energy is infinite and power of the signal is finite.

For a power signal P_g = Finite

and E_g = Infinite

Note: *All the practicals signals are only energy signal because there is no any practical signal whose energy is infinite.*

1.3.4 Periodic and Non-periodic signals

If a signal repeats same waveform after certain time interval then it is called periodic otherwise it is called non-periodic signal.

1.3.5 Deterministics and Random signals

If it is possible to determine the future value of signal at any instant of time from the knowledge of previous values, then it is known as deterministic signal.

Deterministic signal is described by unique mathematical expression.

On the otherhand if a signal is described by probability function only. It is called non-deterministic or random signal.

1.3.6 Even and Odd signal

A signal x(t) is said to be an even signal if,

$$x(-t) = x(t)$$

On the otherhand a signal is said to be an odd signal if,

$$x(-t) = -x(t)$$

If any signal is neither even nor odd signal then it is called neither even nor odd (NENO) signal.

1.3.7 Causal and Non causal signal

A signal x(t) is said to be causal if it is zero for t less than zero

i.e. for x(t) to be causal

$$x(t) = 0 \text{ for } t < 0$$

A signal is said to be anticausal if,

$$x(t) = 0 \text{ for } t > 0$$

If signal is neither causal nor anti causal then it is non-causal signal.

Example: Which of the following signal is causal, anti causal or non causal

(i) $e^{t}\, u(-t)$ (ii) $e^{-t}\, u(t)$ (iii) $\sin t\, u(t-2)$

(iv) s (t–2) (v) $e^{t}\, u(t+2)$ (vi) $e^{t}\, u(t-2)$

(vii) $e^{-t}\, u(-t-2)$

Solution: (i) Anti causal (ii) Causal (iii) Causal

(iv) Causal (v) Non-causal (vi) Causal

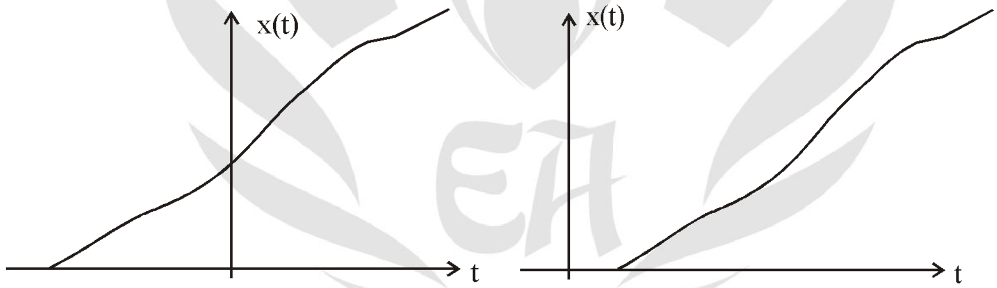
(vii) Anticausal

1.3.8 Right sided and left sided signals

(A) Right sided signals :

A signal extending to $+\infty$ is called right handed signal

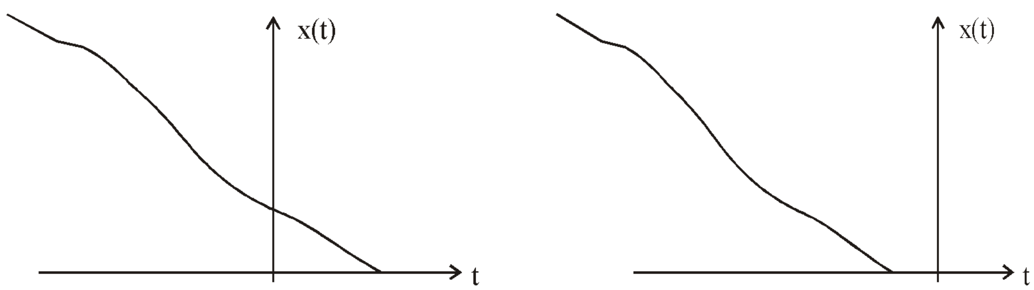
Example:



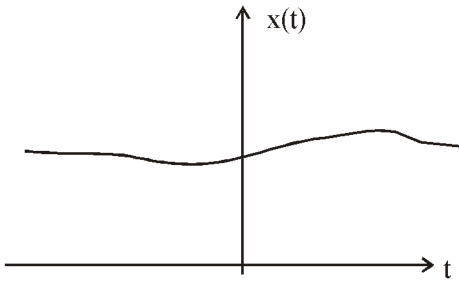
(B) Left sided signals :

A signal extending to $-\infty$ is called left sided signal

Example:



A signal extended from $-\infty$ to $+\infty$ is called etternal signal.



1.4 AREA OF A SIGNAL

Area of a signal in continuous time scale is given as

$$A = \int_{-\infty}^{\infty} x(t).dt$$

And in case of discrete time area is given as,

$$A = \sum_{n=-\infty}^{\infty} x(n)$$

1.4.1 Absolute Value of a Signal :

Absolute value of a signal in continuous time scale is given as

$$|A| = \int_{-\infty}^{\infty} |x(t)|\, dt$$

and incase of discrete time it is given as–

$$|A| = \sum_{n=-\infty}^{\infty} |x[n]|$$

1.5 BASIC OPERATIONS ON SIGNAL

Broadly operations on signal can be classify into two categories :

(i) Operation on amplitude

(ii) Operation on time scale

1.5.1 Operation on Amplitude :

Operation on amplitude can be classify into three categories as :

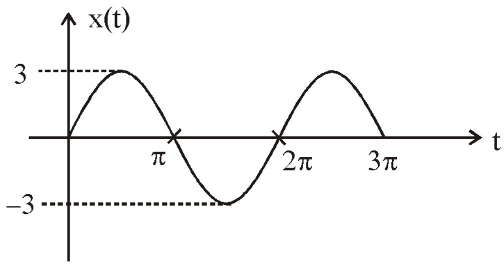
(A) Amplitude shifting or clamping

(B) Amplitude scaling

(C) Amplitude inversion

(A) Amplitude Shifting or Clamping :

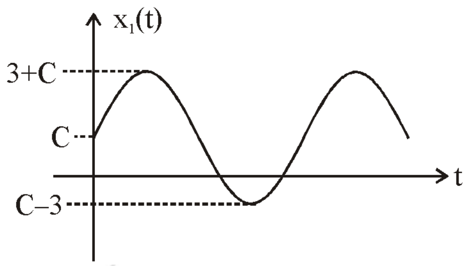
Consider a signal x(t) = 3 sint, as shown in the figure.



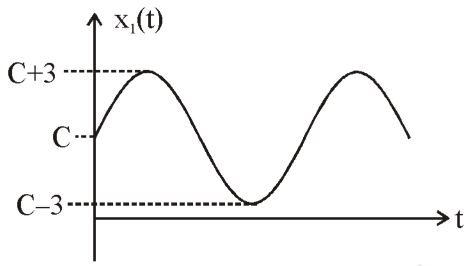
Then $x_1(t) = x(t) \pm C$ is called amplitude shifted version of x(t) i.e.

$$x_1(t) = 3 \sin t \pm C$$

For +, it is called up-shifting i.e. $x_1(t) = 3 \sin t + C$ is the up-shifting version of x(t), as shown in figure.

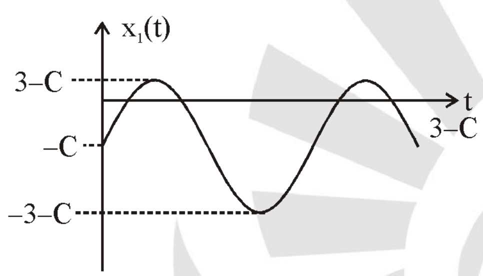


For C < 3

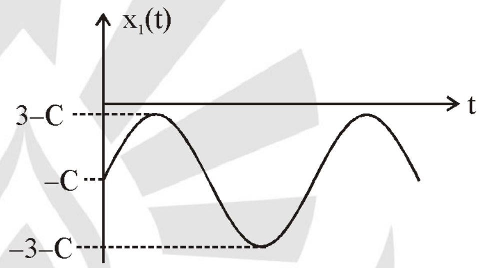


For C > 3

For –, it is called down shifting i.e. $x_1(t) = 3\sin t - C$ is the down-shifting version of x(t) as shown in figure.



For C < 3



For C > 3

(B) Amplitude Scalling :

Multiplication of a signal with any constant K is called amplitude scaling i.e. $x_1(t) = K\, x(t)$ is the amplitude scaled version of x(t)

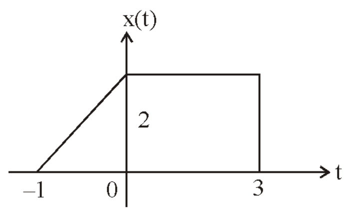
Now if $|K| > 1$ then it is called amplification.

Now if $|K| < 1$ then it is called attenuation.

(C) Amplitude Inversion :

It is special case of scaling in which $K = -1$. It is the folded version of x(t) about horizontal axis.

Example :

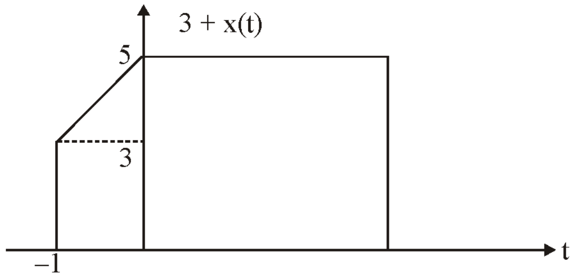


x(t) is shown in the figure sketch the following :

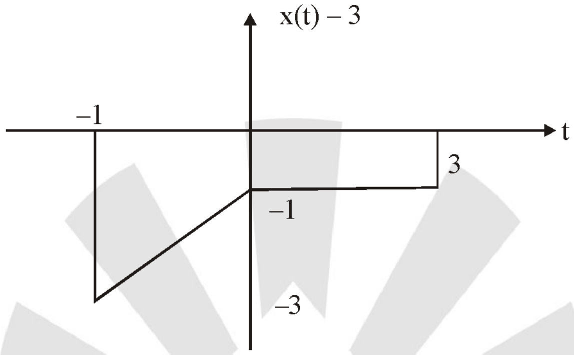
(i) $3 + x(t)$ (ii) $x(t) - 3$ (iii) $2x(t)$

(iv) $\frac{1}{2}x(t)$ (v) $-x(t) + 2$

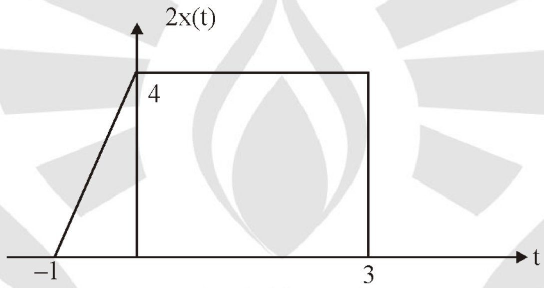
Solution: (i) $3 + x(t)$: It is the amplitude shifting by 3 units



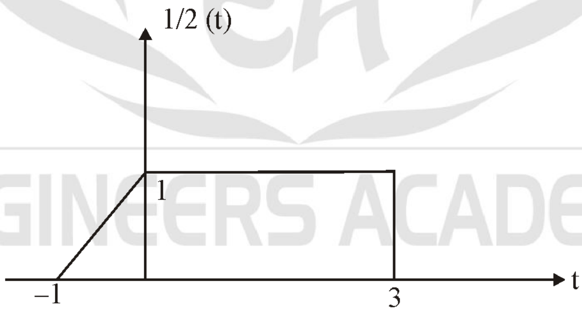
(ii) x(t) – 3 : It is the amplitude shifting by –3 units.



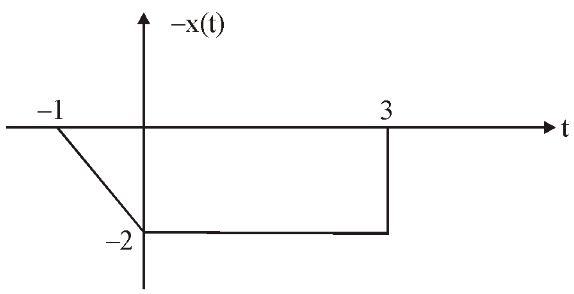
(iii) 2x(t) : it is the amplitude scaling of x(t) by 2 units i.e. it is the amplification of x(t).



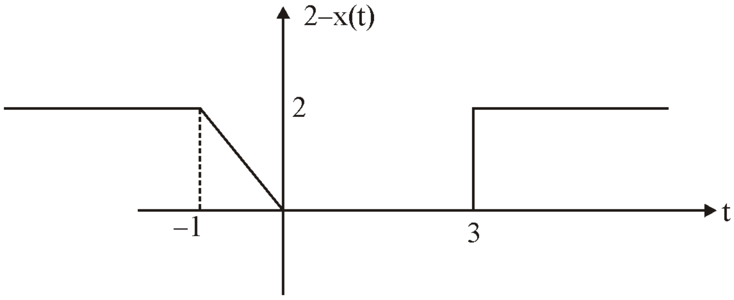
(iv) $\frac{1}{2}x(t)$: It is attenuation of x(t)



(v) –x(t) + 2 : It is a composite operation, in the amplitude operation firstly obtain –x(t) then shift by +2 unit.



Now shift –x(t) by + 2 unit.



1.5.2 Operation on Time Scale :

Operations on time scale can also be classify into three categories :

(A) Time shifting

(B) Time scaling

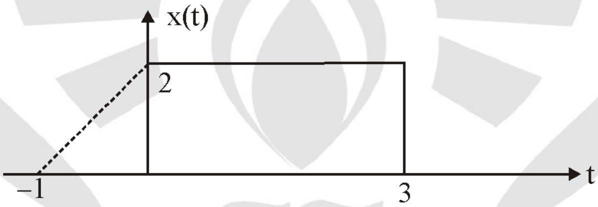
(C) Time inversion

(A) Time Shifting :

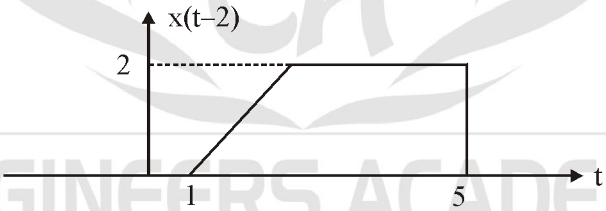
$x\ (t \pm T_0)$ is called time shifted version of x(t). It may results delay or advance.

$x(t - T_0)$ is called delayed version of x(t) and $x(t + T_0)$ is called advanced version of x(t)

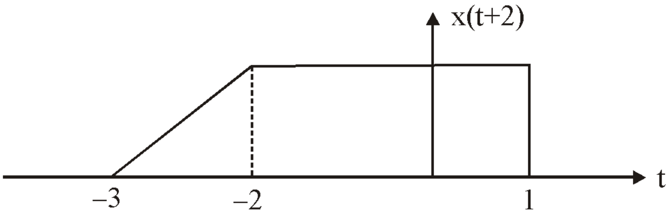
Consider x(t) as shown in the figure.



Then, x(t–2) means shifts right the signal by 2 units.



x(t+2) means shifts left the signal by 2 units.

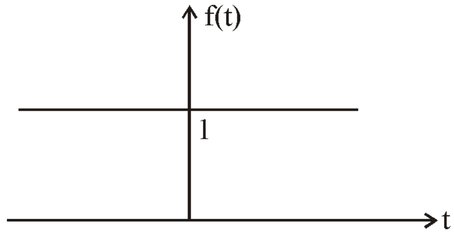


(B) Time Scaling :

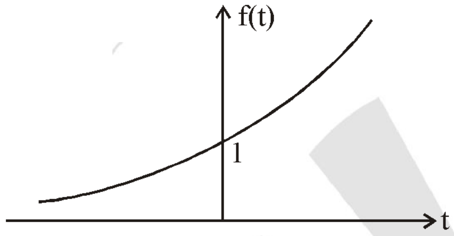
x(at) is the time scaled version of x(t).

Now if |a| > 1 then x(at) is the compressed version of x(t) and if |a| < 1 then x(at) is the expanded version of x(t).

Case 1 : When $\sigma = 0$, and $\omega = 0$, $f(t) = 1$

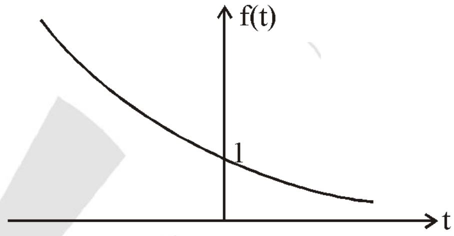


Case 2 : When $\sigma \neq 0$ and $\omega = 0$, $f(t) = e^{\sigma t}$



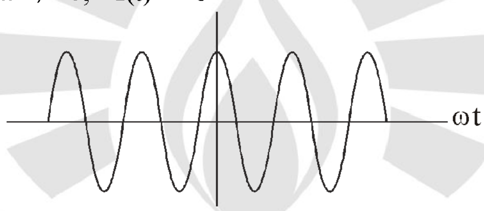
for $\sigma > 0$

and it is called exponentially growing function



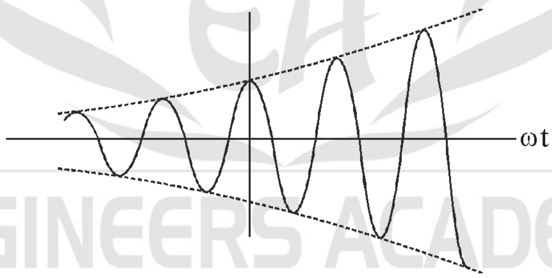
for $\sigma < 0$. It is called exponential decaying function.

Case 3 : When $\sigma = 0$ and $\omega \neq 0$, $f(t) = e^{j\omega t}$

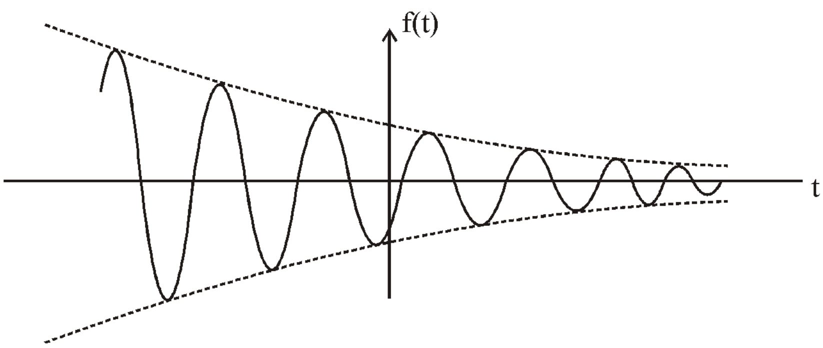


it is called sinusoidal function

Case 4 : When $\sigma \neq 0$ and $\omega \neq 0$, $f(t) = e^{\sigma t} \cdot e^{j\omega t}$, if $\sigma > 0$



and it is called sinusoidal growing signals



if $\sigma < 0$ Sinusoidal decaying function.

1.9 EVEN AND ODD FUNCTION

A signal is said to be an even signal if

$x(-t) = x(t)$ for all value of t and it is called odd if,

$x(-t) = -x(t)$ for all value of t

Any signal that does not satisfy either of the above condition is called neither even nor odd signal, and such types of signal contains some even component and odd component.

A signal can be written as–

$$x(t) = x_e(t) + x_0(t) \quad .... 1$$

Where $x_e(t)$: even component of x(t)

$x_0(t)$: odd component of x(t)

$$x(-t) = x_e(-t) + x_0(-t)$$

$$x(-t) = x_e(t) - x_0(t) \quad .... 2$$

from equation 1 & 2

$$x_e(t) = \frac{1}{2}\{x(t)+x(-t)\}$$

$$x_0(t) = \frac{1}{2}\{x(t)-x(-t)\}$$

Note: *(i) Odd component of an even signal is zero, similarly even component of an odd signal is zero.*

(ii) Even + Even ⇒ Even signal

Odd + Odd ⇒ Odd signal

Odd + Even ⇒ Neigher even nor odd signal

Example: Check whether the given signal is even or odd, and if it is NENO then find the even and odd component of the signal

(i) sin t (ii) cos t (iii) sgn(t)

(iv) u(t) (v) e^{-jt} (vi) e^{jt}

(vii) e^{t} (viii) cos ht (ix) sin ht

Solution : (i) $x(t) = \sin t$

$$= \frac{e^{jt}-e^{-jt}}{2j}$$

$$\Rightarrow \quad x(-t) = \frac{e^{-jt}-e^{jt}}{2j} = -\frac{1}{2j}\{e^{jt}-e^{-jt}\}$$

Since $x(-t) = -x(t)$ therefore

x(t) is an odd function

(ii) $$x(t) = \cos t = \frac{e^{jt}+e^{-jt}}{2}$$

$$x(-t) = \frac{e^{-jt}+e^{jt}}{2} = x(t)$$

Since $x(-t) = x(t)$, therefore x(t) is an even function

(iii) $x(t) = \text{sgn}(t)$

$= u(t) - u(-t)$

$\Rightarrow$ $x(-t) = u(-t) - u(t)$

$= -\{u(t) - u(-t)\}$

$= -x(t)$

Since $x(-t) = -x(t)$, therefore

x(t) is an odd function

(iv) $x(t) = u(t)$

$x(-t) = u(-t)$

Since $x(-t) \neq x(t)$ and also

$x(-t) \neq -x(t)$, therefore

x(t) is NENO function of x(t), there fore even component of x(t),

$$x_e(t) = \frac{1}{2}\{x(t)+x(-t)\}$$

$$= \frac{1}{2}\{u(t)+u(-t)\}$$

$$x_e(t) = \frac{1}{2}$$

and odd component of x(t)

$$x_0(t) = \frac{1}{2}\{x(t)-x(t)\}$$

$$= \frac{1}{2}\{u(t)-u(-t)\}$$

$$x_0(t) = \frac{1}{2}\text{sgn}(t)$$

Therefore,

$$x(t) = x_e(t) + x_0(t)$$

$$u(t) = \frac{1}{2}+\frac{1}{2}\text{sgn}(t)$$

(v) $x(t) = e^{-jt}$

$x(-t) = e^{jt}$

It is NENO function, therefore

$$x_e(t) = \frac{1}{2}\{x(t)+x(-t)\}$$

$$= \frac{1}{2}\{e^{-jt}+e^{jt}\}$$

$$x_e(t) = \cos t$$

and odd component is given as,

$$x_0 = \frac{1}{2}\{x(t)-x(-t)\}$$

$$= \frac{1}{2}\{e^{-jt}-e^{jt}\}$$

$$x_0(t) = -\frac{1}{2}\{e^{jt} - e^{-jt}\}$$

$$x_0(t) = -j \sin t$$

Therefore,

$$e^{-jt} = \cos t - j\sin t$$

(vi) $$x(t) = e^{jt}$$

It is NENO function, therefore,

$$x_e(t) = \frac{1}{2}\{e^{jt} + e^{-jt}\}$$

$$= \cos t$$

and odd component,

$$x_0(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

$$= \frac{1}{2}\{e^{jt} - e^{-jt}\} = j \sin t$$

Therefore, $$e^{jt} = \cos t + j \sin t$$

(vii) $$x(t) = e^{t}$$

$$x(-t) = e^{-t}$$

It is NENO function, therefore

$$x_e(t) = \frac{1}{2}\{e^{t} + e^{+t}\}$$

$$x_e(t) = \cos ht$$

and $$x_0(t) = \frac{1}{2}\{e^{t} - e^{-t}\} = \sin ht$$

Therefore,

$$e^{t} = \cosh t + \sin ht$$

(viii) $$x(t) = \cos ht$$

$$= \frac{e^{t} + e^{-t}}{2}$$

$$\Rightarrow \quad x(-t) = \frac{e^{-t} + e^{t}}{2} = x(t)$$

Since, $x(-t) = x(t)$, therefore

it is an even function

(ix) $$x(t) = \sin ht$$

$$= \frac{e^{t} - e^{-t}}{2}$$

$$\Rightarrow \quad x(-t) = \frac{e^{-t} - e^{t}}{2}$$

$$= \frac{(e^{t} - e^{-t})}{2} = -x\ (t)$$

Since $x(-t) = -x(t)$, therefore $x(t)$ is an odd function.

Example: Check whether the following function is even or odd, and also determine the even and odd component of the function

(i) $4t^4 - 3t^3 + 2t^2 + t + 1$

(ii) sin ht. cosht

(iii) sin t + cos 2t + sin 3t

(iv) e^{-2t} . cos3t

Solution: (i)

$$x(t) = 4t^4 - 3t^3 + 2t^2 + t + 1$$

$$x(-t) = 4t^4 + 3t^3 + 2t^2 - t + 1$$

It is NENO function

Therefore,

$$x_e(t) = \frac{1}{2}\{x(t)+x(-t)\}$$

$$= \frac{1}{2}\{4t^4 - 3t^3 + 2t^2 + t + 1 + 4t^4 + 3t^3 + 2t^2 - t + 1\}$$

$$x_e(t) = 4t^4 + 2t^2 + 1$$

and odd component,

$$x_0(t) = \frac{1}{2}\{x(t)-x(-t)\}$$

$$= \frac{1}{2}\{4t^4 - 3t^3 + 2t^2 + t + 1 - 4t^4 - 3t^3 - 2t^2 + t - 1\}$$

$$x_0 = -3t^3 + t$$

(ii)

$$x(t) = \text{sin ht. cosht}$$

$$x(t) = x_1(t)\ .\ x_2(t)$$

Since, $x_1(t)$ is and odd function and $x_2(t)$ is an even function.

So, x(t) is an odd function

Note:

odd × odd = Even

odd × Even = Odd

Even × Even = Even

(iii)

$$x(t) = \sin t + \cos 2t + \sin 3t$$

$$x(t) = x_1(t) + x_2(t) + x_3(t)$$

Here, $x_1(t)$ is an odd function

$x_2(t)$ is an even function

$x_3(t)$ is an odd function

as we know that,

Even + odd $\Rightarrow$ NENO function

Therefore, x(t) is NENO function

Now odd component of x(t)

$$x_0(t) = x_1(t) + x_3(t)$$

$$x_0(t) = \sin t + \sin 3(t)$$

and even component of x(t)

$$x_e(t) = x_2(t)$$
$$= \cos 2t$$

(iv) $$x(t) = e^{-2t} . \cos 3t$$
$$x(t) = [\cosh(2t) - \sinh(2t)] \cos 3t$$
$$= \underbrace{\cos 2ht}_{\text{Even}} \times \underbrace{\cos 3t}_{\text{Even}} - \underbrace{\sin 2ht}_{\text{odd}} \times \underbrace{\cos 3t}_{\text{odd}}$$

Even* – *Odd

$$x(t) = \text{NENO function}$$

Therefore, Even component of x(t)

$$x_e(t) = \cos 2ht. \cos 3t$$

and odd component of x(t)

$$x_0(t) = -\sin 2ht . \cos 3t$$

Example: Determine the even and odd part of x(n) = u(n)

Solution:

$$x(n) = u(n)$$
$$x(-n) = u(-n)$$

Even part of x(n) $$x_e(n) = \frac{1}{2}\{x(n) + x(-n)\} = \frac{1}{2}\{u(n) + u(-n)\}$$

$$x_e = \frac{1}{2}\{1 + \delta(n)\}$$

and odd part of x(n) $$x_0(n) = \frac{1}{2}\{x(n) - x(-n)\}$$

$$= \frac{1}{2}\{u(n) - u(-n)\} = \frac{1}{2}\text{sgn}(n)$$

Therefore, $$u(n) = \frac{1}{2}\{1 + \delta(n) + \text{sgn}(n)\}$$

1.10 PERIODIC AND NON-PERIODIC SIGNALS

In Continuous Time :

A signal x(t) is said to be periodic if,

$$x(t) = x(t + T)$$

If any signal does not satisfy above condition, it is called non-periodic signal

Note: *The above definition is valid except if x(t) is constant, because in this case it is periodic for any value of T, and hence its fundamental period is undefined.*

1.10.1 Necessary Condition For Periodicity :

Every periodic signal repeats it self from $-\infty$ to ∞, therefore periodic signal must be etternal.

Example: Check the periodicity of following signals

(i) $3\sin\left(\frac{\pi}{3}t\right)$ (ii) $\sin(\pi t)\,u(t)$ (iii) $-5\cos\sqrt{3}\,t$

(iv) $4\,e^{-5t}$ (v) $2\sin^2\left(\frac{\pi}{2}t\right)$

Solution : (i) $3\sin\left(\frac{\pi}{3}t\right)$

Here, $\omega_0 = \frac{\pi}{3}$,

There for, $T = \frac{2\pi}{\omega_0}$

$$T = \frac{2\pi}{\pi/3} = 6$$

(ii) $\sin(\pi t)\,u(t)$

It is non-periodic since it is not defined from $-\infty$ to $+\infty$

(iii) $-5\cos\sqrt{3}t$

Here, $\omega_0 = \sqrt{3}$, $T = \frac{2\pi}{\sqrt{3}}$

(iv) $4e^{-5t} \Rightarrow$ It is non-periodic

(v) $x(t) = 2\sin^2\left(\frac{\pi}{3}t\right) = 1 - \cos\frac{2\pi}{3}t$

So, $\omega_0 = \frac{2\pi}{3}$

$$T = \frac{2\pi}{2\pi/3} = 3$$

1.10.2 Fundamental Period of Composite Signal :

Let a composite signal is defined as,

$$x(t) = x_1(t) \pm x_2(t) \pm x_3(t) \pm x_4(t)$$

Then to determine the period of x(t) following procedure should be following :

Step 1 : Find the fundamental period of each signal $x_1(t)$, $x_2(t)$ ....Let these periods are T_1, T_2, ...

Step 2 : Find the ratio of T_1/T_2, T_2/T_3, T_3/T_4.

Step 3 : If all the ratios are rational, then x(t) will be periodic.

Step 4 : If any one of the ratio is irrational then x(t) is non-periodic

Step 5 : If x(t) is periodic then its fundamental period is given as

$$T = \frac{\text{LCM of numerator of } (T_1, T_2, T_3 ....)}{\text{HCF of denominator of } (T_1, T_2 T_3 ......)}$$

Example: Check the periodicity and hence find the fundamental period of given signals:

(i) $\cos 4t + \sin 3t$

(ii) $\cos t \sin 3t$

(iii) e^{-2jt}

(iv) $\sin 3t + \cos\left(\frac{\pi}{5}\right)t$

(v) $\cos 3\pi t + \cos\left(\frac{2\pi}{3}\pi\right)t$

(vi) $\sin\sqrt{3}t + \cos 3t$

(vii) $\sin\frac{\pi}{3}t + 5\sin(0.3\pi t) - 10\cos\ 10\pi t$

(viii) $j\ e^{-j2t}$

Solution : (i)

$$x(t) = \cos 4t + \sin 3t$$

$$T_1 = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$T_2 = \frac{2\pi}{3} = \frac{2\pi}{3}$$

$$\frac{T_1}{T_2} = \frac{\pi/2}{2\pi/3} = \frac{3}{2} \text{ (Rational)}$$

There x(t) is periodic and period is given as

$$T = \frac{\text{LCM of } \{\pi, 2\pi\}}{\text{HCF of } \{2, 3\}}$$

$$T = \frac{2\pi}{1} = 2\pi$$

(ii) $\cos t \cdot \sin 3t \Rightarrow \frac{1}{2}\{\sin 4t + \sin 2t\}$

$$T_1 = \frac{2\pi}{4}, \frac{\pi}{2}, T_2 = \frac{2\pi}{2} = \pi$$

Now, $$\frac{T_1}{T_2} = \frac{\pi/2}{\pi} = \frac{1}{2} \text{ (Rational)}$$

Therefore x(t) is periodic and its period is given as,

$$T = \frac{\text{LCM of } \{\pi, \pi\}}{\text{HCF of } \{1,2\}} = \pi$$

$$T = \pi$$

(iii) e^{-2jt}

$$= \cos 2t - j \sin 2t$$

$$T_1 = \frac{2\pi}{2} = \pi$$

$$T_2 = \frac{2\pi}{2} = \pi$$

$$\frac{T_1}{T_2} = 1 \text{ (Rational)}$$

So, $T = \pi$

(iv) $\sin 3t + \cos\left(\frac{\pi}{5}\right)t$

$$T_1 = \frac{2\pi}{3},\ T_2 = \frac{2\pi}{\pi/5} = 10$$

$$\frac{T_1}{T_2} = \frac{2\pi}{3\times 10} \Rightarrow \text{(Irrational)}$$

Therefore ($\sin 3t + \cos\frac{\pi}{5}t$) is non periodic

Note: *π is an irrational number*

(v) $\cos 3\pi t + \cos\left(\frac{2\pi}{3}\right)t$

$$T_1 = \frac{2\pi}{2\pi} = 1$$

$$T_2 = \frac{2\pi}{2\pi/3} = 3$$

$$\frac{T_1}{T_2} = \frac{1}{3} \text{(Rational)}$$

Therefore x(t) is periodic,

$$T = \frac{\text{LCM of }\{1,3\}}{\text{HCF of }\{1,1\}} = 3$$

$$T = 3$$

(vi) $\sin\sqrt{3}t + \cos 3t$

$$T_1 = \frac{2\pi}{\sqrt{3}},\ T_2 = \frac{2\pi}{3}$$

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{3}}\times\frac{3}{2\pi} = \sqrt{3} \text{ (Irrational)}$$

Therefore x(t) is non-periodic

(vii) $\sin\frac{\pi}{3}t + 5\sin(0.3\pi t) - 10\cos 10\pi t$

$$T_1 = \frac{2\pi}{\pi/3} = 6$$

$$T_2 = \frac{2\pi}{0.3\pi} = \frac{20}{3}$$

$$T_3 = \frac{2\pi}{10\pi} = \frac{1}{5}$$

Now check the ratios,

$$\frac{T_1}{T_2} = \frac{6}{20/3} \Rightarrow \frac{6\times 3}{20} = \frac{9}{10} \text{ (Rational)}$$

$$\frac{T_1}{T_3} = \frac{20/3}{-15} \Rightarrow \frac{100}{3} \text{ (Rational)}$$

Therefore x(t) is periodic, $T = \frac{\text{LCM}\{6,20,1\}}{\text{HCF}\{1,3,5\}} = 60$

(viii) je^{-2jt}

$$j\{\cos 2t - j \sin 2t\}$$

$$T_1 = \frac{2\pi}{2} = \pi,\ T_2 = \frac{2\pi}{2} = \pi$$

$$T = \frac{\text{LCM}\{\pi,\pi\}}{\text{HCF}\{1,1\}} = \pi$$

1.10.3 Periodicity of Discrete Time Signal :

Discrete time signal is said to be periodic,

If $x(n) = x(n+N)$

or $x(n) = x(n+mN)$

where, m : integer

Let $x(n) = \cos(\omega n + \theta)$ ....(1)

then, $x(n+1) = \cos(\omega n + \theta + \omega N)$ ....(2)

If x(n) is periodic, then equation (1) must be same as equation (2)

i.e. $\omega N = 2\pi K$

$$N = \left(\frac{2\pi}{\omega}\right)K$$

Note: *(i) Fundamental period of continuous signal can be integer or non-integer but in case of discrete time period is always an integer.*

(ii) In discrete time for a signal to be periodic is must be multiple of π.

In continuous time periodicity of composite signal is not guaranteed whereas in discrete time if individual signal is periodic then their composite signal is always periodic.

Example: Determine the periodicity of the following signals.

(i) $x(n) = 4\cos\left(\frac{4}{3}\pi n\right) u(n)$ (ii) $x(n) = 4\cos\left(\frac{\pi}{3}n\right)$

(iii) $x(n) = \sin^2(0.2\pi n)$ (iv) $x(n) = \sin 4n$

(v) $x(n) = \sin\frac{\pi}{4}n + \cos 3n$ (vi) $x(n) = \sin 5\pi n + \sin\frac{\pi}{0.3}n$

Solution : (i) $x(n) = 4\cos\left(\frac{4}{3}\pi n\right) u(n)$

Since it is not defined from $-\infty$ to $+\infty$ therefore it is non-periodic.

(ii) $x(n) = 4\cos\left(\frac{\pi}{3}n\right)$

$$\omega = \frac{\pi}{3},$$

$$N = \left(\frac{2\pi}{\pi/3}\right)K$$

$$N = 6K$$

$$N = 6\ \{K = 1\}$$

Note: *Always K is the value of lowest possible integer, that makes N integer.*

(iii) $x(n) = \sin^2(0.2\pi n)$

$$= \frac{1}{2} - \frac{1}{2}\cos(0.4\pi n)$$

$\omega = 0.4\pi,$

$$N = \left(\frac{2\pi}{\omega}\right)K$$

$$N = \left(\frac{2\pi}{0.4\pi}\right)K = 5K$$

$N = 5 \quad (K = 1)$

(iv) $x(n) = \sin 4n$

$\omega = 4,$

$$N = \left(\frac{2\pi}{\omega}\right)K$$

$$N = \left(\frac{\pi}{2}\right)(K)$$

N can not be integer for any value of K, therefore x(n) is non-periodic

(v) $x(n) = \sin\frac{\pi}{4}n + \cos 3n$

$$\omega_1 = \frac{\pi}{4},\ N_1 = \left(\frac{2\pi}{\pi/4}\right)K = 8K$$

$N_1 = 8 \ (K = 1)$

$$\omega_2 = 3,\ N_2 = \left(\frac{2\pi}{\omega^2}\right)K$$

$$N_2 = \left(\frac{2\pi}{3}\right)K$$

Since, N_2 can not be integer for any value of K, therefore, cos 3n is non-periodic, Hence overall signal x(n) is non-periodic.

(vi) $x(n) = \sin 5\pi n + \sin\frac{\pi}{0.3}n$

$$N_1 = \left(\frac{2\pi}{5\pi}\right)K$$

$$= \frac{2}{5}K = 2 \ (\text{for } K = 5)$$

$$N_2 = \left(\frac{2\pi}{\pi\ 10.3}\right)K = 0.6\ K = 3\ \{\text{for } k = 5\}$$

Therefore period of x(n) = LCM of (2, 3)

$N = 6$

1.11 ENERGY AND POWER OF A SIGNAL

In continuous Time :

$E = \int_{-\infty}^{\infty} x^2(t).dt$: for real valued signal x(t)

$E = \int_{-\infty}^{\infty} |x(t)|^2 .dt$: for complex value signal x(t)

In discrete Time :

$E = \sum_{-\infty}^{\infty} x^2(n)$: for real valued signal x(n)

$E = \sum_{-\infty}^{\infty} |x(n)|^2$: for complex values signal x(n).

Unit :

This is normalized energy, therefore its unit is not joule, its unit is depends on x(t) if x(t) voltage signal, then unit of E is volt^2 second, if x(t) is current signal then unit of E is Amp^2. sec.

Example: Find the energy of given signals :

(i) Au(t)

(ii) r(t)

(iii) e^{-at} u(t) a > 0

(iv) e^{at} u(–t) a > 0

(v) $(e^{-at} + e^{-bt})$ u(t) a and b > 0

(vi) A $e^{j\omega_0 t}$

(vii) A sin (ωt + θ)

(viii) A cos (ωt + θ)

Solution : (i) $x(t) = Au(t)$

$$E = \int_0^{\infty} A^2.dt = A^2(t)_0^{\infty} = \infty$$

So, $E = \infty$

(ii) $x(t) = r(t) = t\,u(t)$

$$E = \int_0^{\infty} (t)^2 dt = \frac{1}{3}\left(t^3\right)_0^{\infty} = \infty$$

$E = \infty$

(iii) $x(t) = e^{-at}\,u(t)$

$$E = \int_0^{\infty} (e^{-at})^2 dt = \int_0^{\infty} (e^{-at})^2 dt$$

$$= \int_0^{\infty} e^{-2at}.dt = -\frac{1}{2a}\left[e^{-2at}\right]_0^{\infty}$$

$$E = -\frac{1}{2a}\left\{e^{-\infty} - e^0\right\} = \frac{1}{2a}\{0-1\}$$

$$E = \frac{1}{2a}$$

(iv) $x(t) = e^{at}\, u(-t)$

$$E = \int_{-\infty}^{0} \left(e^{at}\right)^2 .dt \qquad \int_{-\infty}^{0} e^{2at}.dt$$

$$= \frac{1}{2a}\left[e^{2at}\right]_{-\infty}^{0}$$

$$E = \frac{1}{2a}$$

(v) $x(t) = (e^{-at} + e^{-bt})\, u(t)$

$$E = \int_{0}^{\infty} \left(e^{-at} + e^{-bt}\right)^2 dt$$

$$= \int_{0}^{\infty} \left(e^{-2at} + e^{-2bt} + 2e^{-(a+b)t}\right) dt$$

$$E = \frac{1}{2a} + \frac{1}{2b} + \frac{2}{a+b}$$

(vi) $x(t) = A\,e^{j\omega t}$

$$|x(t)| = A$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 -dt = \int_{-\infty}^{\infty} A^2.dt = \infty$$

(vii) $x(t) = A \sin(\omega t + \theta)$

$$E = \int_{-\infty}^{\infty} A^2.\cos^2(\omega t + \theta).dt$$

$$= \int_{-\infty}^{\infty} \frac{A^2}{2} + \frac{A^2}{2}\cos(2\omega t + 2\theta)dt$$

$$= \int_{-\infty}^{\infty} \frac{A^2}{2}dt + \int_{-\infty}^{\infty} \frac{A^2}{2}\cos(2\omega t + 2\theta)\ dt$$

$$= \infty \quad + \quad 0$$

$$= \infty$$

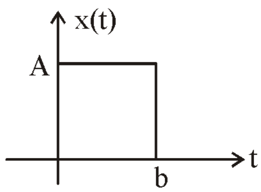
$$E = \infty$$

(viii) $x(t) = A \cos(\omega t + \theta)$

Similarly it is ∞ energy

Note: *Irrespective of the frequency, phase, and amplitude sinusoidal signal of infinite duration always have ∞ energy.*

Example : Find the energy of given pulse x(t)

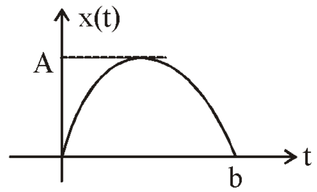


Solution :

$$E = \int_0^b A^2 dt$$

$$E = A^2.b$$

Example: Find the energy of given pulse x(t) = A sin (πt/b)



Solution :

$$x(t) = A \sin \frac{\pi}{b}t \quad 0 < t < b$$

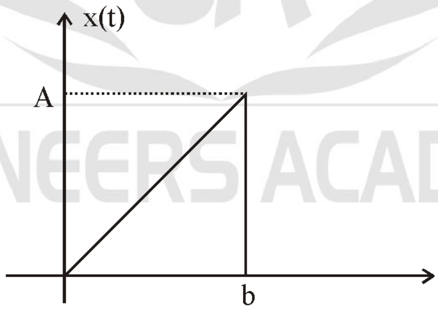
$$E = \int_0^b A^2 \sin^2 \frac{\pi}{b}t = \int_0^b \frac{A^2}{2}\left(1 - \cos\frac{2\pi}{b}t\right) dt$$

$$= \frac{A^2}{2}\left[t - \sin\frac{2\pi t}{b}\right]_0^b = \frac{A^2}{2}\{(b-0)-(0-0)\}$$

$$= \frac{A^2}{2}b$$

$$E = \frac{A^2 b}{2}$$

Example: Find the energy of given pulse :



Solution :

$$E = \int_0^b \left(\frac{A}{b}t\right)^2 = \frac{A^2}{b^2} \times \frac{1}{3}\{t3\}_0^b$$

$$E = \frac{A^2 b}{3}$$

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PRACTICE SHEET

OBJECTIVE QUESTIONS

1. Which of the following signals is/ are periodic?

(a) $x(t) = \cos 2t + \cos 3t + \cos 5t$

(b) $x(t) = \exp(j8\pi t)$

(c) $x(t) = \exp(-7t) \sin 10\pi t$

(d) $x(t) = \cos 2t \cos 4t$

2. ***Assertion (A) :*** An LTI discrete system represented by the difference equations $y(n + 2) - 5y(n + 1) + 6y(n) = x(n)$ is unstable

Reason (R): A system is unstable if the roots of the characteristic equation lie outside the unit circle.

(a) Both A and R are true and R is the correct explanation of A

(b) Both A and R are true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

3. Consider a random sinusoidal signal $x(t) = \sin(\omega_0 t + \phi)$ where a random variable 'ϕ' is uniformly distributed in the range $+\pi/2$. The mean value of x(t) is

(a) zero (b) $\frac{2}{\pi}\sin(\omega_0 t)$

(c) $\frac{2}{\pi}\cos(\omega_0 t)$ (d) $\frac{2}{\pi}$

4. The function $\delta(2n)$ is equal to

(a) $\delta(n)$ (b) $\frac{1}{2}\delta(n)$

(c) $2\delta(n)$ (d) $2\delta\left(\frac{n}{2}\right)$

5. Let $\delta(t)$ denote the delta function. The value of the integral $\int_{-\infty}^{\infty} \delta(t)\cos\left(\frac{3t}{2}\right)dt$ is

(a) 1 (b) -1

(c) 0 (d) $\frac{\pi}{2}$

6. If a signal f(t) has energy E, the engergy of the singal f(2t) is equal to

(a) E (b) E / 2

(c) 2E (d) 4E

7. If a function f(t) u(t) is shifted to right side by t_0, then the function can be expressed as

(a) $f(t-t_0)u(t)$ (b) $f(t)u(t-t_0)$

(c) $f(t-t_0)u(t-t_0)$ (d) $f(t+t_0)u(t-t_0)$

8. The color T.V. picture signal is a

(a) Single-channel, one-dimensional signal

(b) single-channel, three dimensional signal

(c) three-channel, one-dimensional signal

(d) three-channel, three-dimensional signal

9. Consider the signals $x_1(t) = 2\sin \pi t + \cos 4\pi t$ and $x_2(t) = 2\sin 5\pi t + 3\sin 13\pi t$ -

(a) Both the signals are periodic

(b) Both the signals are not periodic

(c) x_1 is periodic, but x_2 is not periodic

(d) x_1 is not periodic, but x_2 is periodic

10. The sum of two or more arbitrary sinusoids is

(a) always periodic

(b) Periodic under certain conditions

(c) Never periodic

(d) Periodic only if all the sinusoids are identical in frequency and phase

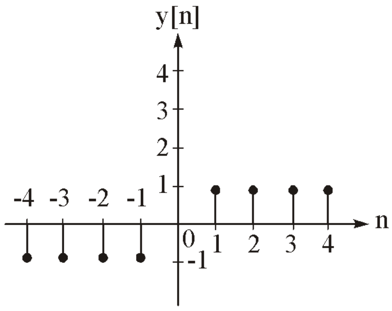
11. Which one of the following must be satisfied if a signal is to be periodic for $-\infty < t < \infty$?

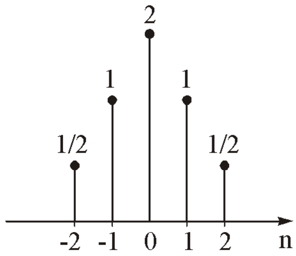
(a) $x(t+T_0) = x(t)$

(b) $x(t+T_0) = dx(t)/dt$

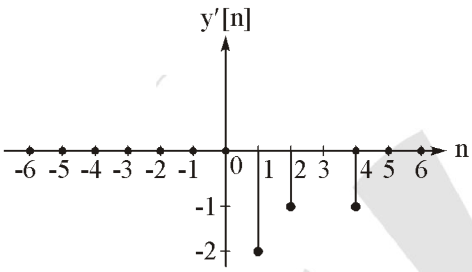
(c) $x(t+T_0) = \int_{t}^{T_0} x(t)\,dt$

(d) $x(t+T_0) = x(t) + kT_0$





the signal



represents.

(a) $y'[n] = x[n-3] \cdot y[-n]$

(b) $y'[n] = x[3-n] \cdot y[-n]$

(c) $y'[n] = x[-n-3] \cdot y[-n]$

(d) $y'[n] = x[3-n] \cdot y[-n]$

49. The signal $x(t) = A\cos(\omega t + \phi)$ is

(a) an energy signal

(b) a power signal

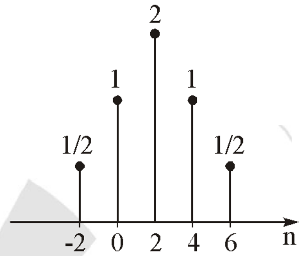
(c) an energy as well as a power signal

(d) neither an energy nor a power signal

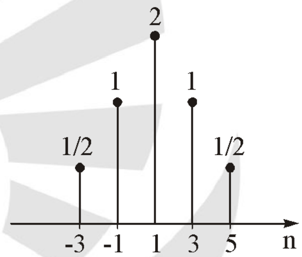
50. A sequency x(n) has non-zero values as shown in figure The sequence

$$y(n) = \left\{ x\left(\frac{n}{2} - 1\right) \right. \quad \text{for 'n' even}$$
$$= 0, \text{for 'n' odd}$$

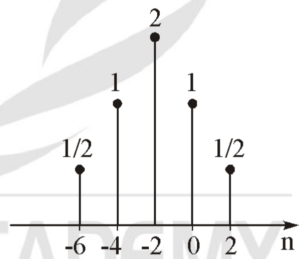
(a)



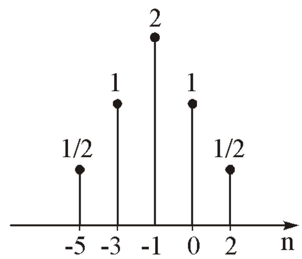
(b)



(c)



(d)



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ANSWERS AND EXPLANATIONS

1. ***Ans. (a), (b), (c)***

(i) $x(t) = \cos 2t + \cos 3t + \cos 5t$

$x(t) = x_1(t) + x_2(t) + x_3(t)$

Where, $x_1(t) = \cos 2t = \cos \omega_1 t$

$\Rightarrow \quad \omega_1 = 2 = \frac{2\pi}{T_1}$ and $T_1 = \pi$

and $x_2(t) = \cos 3t = \cos \omega_2 t$

$\Rightarrow \quad \omega_2 = 3 = \frac{2\pi}{T_2}$ and $T_2 = \frac{2\pi}{3}$

and $x_3(t) = \cos 5t = \cos \omega_3 t$

$\Rightarrow \quad \omega_3 = 5 = \frac{2\pi}{T_3}$ and $T_3 = \frac{2\pi}{5}$

Ratio of time periods,

$$\frac{T_1}{T_2} = \frac{\pi}{2\pi/3} = \frac{3}{2} = 1.5 = \text{Rational number}$$

$$\frac{T_1}{T_3} = \frac{\pi}{2\pi/5} = \frac{5}{2} = 2.5 = \text{Rational number}$$

$$\frac{T_2}{T_3} = \frac{2\pi/3}{2\pi/5} = \frac{5}{3} = 1.666 = \text{Rational number}$$

Since ratios of time periods of signals are rational numbers so given signal is periodic.

Fundamental period,

$$T_o = \frac{\text{L.C.M. of numerator of } T_1, T_2 \,\&\, T_3}{\text{H.C.F. of denominator of } T_1, T_2 \,\&\, T_3}$$

$$\Rightarrow \quad T_o = \frac{\text{L.C.M. of}(2\pi, 2\pi, 2\pi)}{\text{H.C.F. of}(1,3,5)} = \frac{2\pi}{1} = 2\pi$$

(ii) $x(t) = e^{j8\pi t} = e^{j\omega_o t}$

where, $\omega_o = \frac{2\pi}{T_o} = 8\pi$

$\Rightarrow \quad \boxed{T_o = \frac{1}{4}}$

So, signal is periodic.

(iii) $x(t) = e^{-7t}\sin 10\pi t$

Exponential decaying signals are non-periodic signals.

(iv) $x(t) = \cos 2t \cos 4t$

$$= \frac{1}{2}\left[\cos\left(\frac{2+4}{2}\right)t + \cos\left(\frac{4-2}{2}\right)t\right]$$

$$= \frac{1}{2}[\cos 3t + \cos t] = \frac{1}{2}\cos 3t + \frac{1}{2}\cos t = x_1(t) + x_2(t)$$

where, $x_1(t) = \frac{1}{2}\cos 3t = \frac{1}{2}\cos\omega_1 t$

$\Rightarrow \quad \omega_1 = \frac{2\pi}{T_1} = 3 \;\&\; T_1 = \frac{2\pi}{3}$

and $x_2(t) = \frac{1}{2}\cos t = \frac{1}{2}\cos\omega_2 t$

$\Rightarrow \quad \omega_2 = \frac{2\pi}{T_2} = 1 \;\&\; T_2 = 2\pi$

Ratio of time periods,

$$\frac{T_1}{T_2} = \frac{2\pi/3}{2\pi} = \frac{1}{3} = \text{Rational number}$$

Since ratio of time periods is rational number so signal is periodic. Fundamental period of x(t),

$$T_o = \frac{\text{L.C.M. of}(2\pi, 2\pi)}{\text{H.C.F. of}(1,3)} = \frac{2\pi}{1} = 2\pi$$

2. ***Ans.(a)***

3. ***Ans.(b)***

4. ***Ans. (b)***

5. ***Ans. (a)***

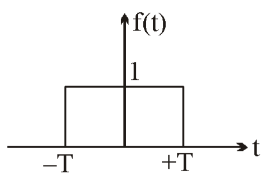
According to sampling property of impulse function,,

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_o)dt = x(t_o)$$

if $x(t) = \cos\frac{3}{2}t$ and $t_o = 0$

then, $\int_{-\infty}^{\infty} \cos\left(\frac{3}{2}t\right)\delta(t)dt = \cos\frac{3}{2}(0) = 1$

6. ***Ans. (b)***



Let $f(t) = 1 \quad ; \quad -T < t < T$

$= 0 \quad ‘ \quad |t| > T$