THEORY & OBJECTIVE PRACTICE BOOKLET

THEORY OF MACHINE

By
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THEORY OF MACHINE

THEORY

1. MACHANISM :

If a number of bodies assembled in such a way that the motion of one causes constrained and predictable motion to others, it is known as 'Mechanism'.

It transmit or modify a motion e.g. slider-crank mechanism, type writer, spring toys.

Machine:

A machine is a machanism or a combination of mechanism which, apart from imparting motion to the part, also transmits and modifies the available mechanical energy into some kind of desired work.

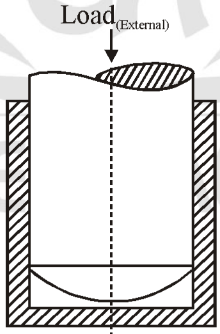
Type of Constrained Motion

Completely constrained motion

When motion between two elements of a pair is in a definite (*single*) direction irrespective of the direction of force applied. It is known as completely constrained motion.

Succesfully constrained motion

When motion between two element of pair is *possible* in *more than one* direction but is made to have motion *only in one* direction by using some *external* means, it is called succesfully constrained motion e.g.



Foot Step Bearing

A pistion in a cylinder of an internal combustion engine is made to have only reciprocating motion due to constrain of the pistion pin (external), cam and follower, shaft in foot step bearing.

Incompletely constrained motion:

When the motion between the elements of a pair is possible in *more than one direction* and depends upon the *direction of force applied,* it is known as incompletely constrained motion e.g. cylindrical shaft in round bearing.

Ridig, Resistant Body

Rigid body:

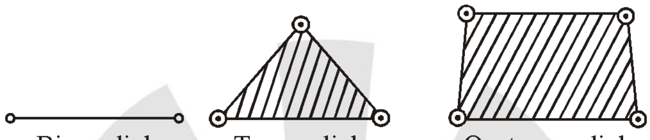
It does not suffer any distortion, under the action of force.

Resistant body:

Those body which are rigid for the purouse they have to serve for e.g. belt drive, where belt is rigid when subjected to tensile forces. Resistant bodies transmit the requried forces with negligivble deformation.

Link

A link id defined as a member of machanism, connecting other member and having motion relative to them.



Classification of Kinematic Pair

According to nature of contact:

- Lower pair:

  A pair of links having *surface or area* contact between the member. e.g. nut turning on a screw, shaft fotating in a bearing, all pair of slider-crank machanism.

- Higher pair

  When a pair has point or line contact between the links, it is known as higher pair.

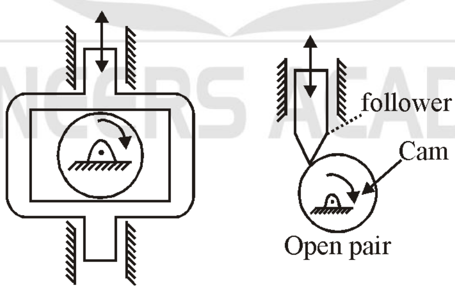
  e.g. wheel rolling on a surface, cam and follower pair.

According to Nature of Machanical Constraint

- Closed pair

  When the element of pair held *mechanically,* it is known as closed pair.

  The contact between the two can be broken by only destruction of *at least one* of the member.



Point to remember:

- All lower pairs are closed pair.
- Open (unclosed pair)

  When two links of a pair are in contact either due to force of gravity or some spring action.

  e.g. cam follower.

According to Nature of Relative Motion

- Sliding pair

  If two links have sliding motion relative to each other, they form sliding pair e.g.

  A rectangular rod in a rectangular hole in a prism.

- Turning pair

  When one link has turning or revolving motion relative to the other, they constitute a turning or revolving pair e.g. curcular shaft revolving in a bearing.

- Rolling pair

  when the link of pair have rolling motion relative to each other, they form a rolling pair, e.g.

  Rolling wheel on flat surface, ball and roller bearing.

- Screw pair (helical)

  It two mating link have turning as well as sliding motion between them form a screw pair e.g. lead screw, nut of lathe.

- Spherical pair

  When one link in the form of a sphere turns inside a fixed link, it is a spherical pair e.g. ball in socket.

Type of Joints

There are three typer of joint.

- Binary joint (B)

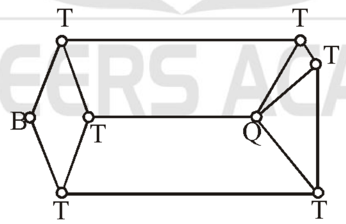
  If two links are joined at the same connection.

- Ternary joint (T)

  if three link are joined at a connection, it is consider equivalent to two binary joint. Since fixing of any one link constitutes two binary joints.

- Quaternary joint (Q)

  If four links are joined at a connection. It is quaternary joint. It is equivalent to three binary joint.



Point to remember:

- If n number of links are connected at a joint, it is equivalent to (n-1) binary joints.

Degree of freedom

The connection of link with another imposes certain constrains on relative motion thus,

Degree of freedom = 6 – number of restraints

Points to remember:

- Number of restraints can never be zero (joint disconnceted)
- Number of restraints can never be 6 (joint become rigid.)

Degree of Freedom of Space Machanism (3-D)

$F = 6 (L - 1) - 5P_1 - 4P_2 - 3P_3 - 2P_4 - P_5$

Here,

F = Degree of freedom (D.O.F.)

L = Total number of links in machanism

P_1 = Number of pair having one D.O.F.

P_2 = Number of pair having two D.O.F, and so on.

Degree of freedom of plane (2D) machanism (Grueble Criterion)

$F = 3 (L - 1) - 2P_1 - P_2$

Here, L = Number of link in a machanism

P_1 = Number of pair shaving one degree of freedom.

P_2 = Number of pair shaving two degree of freedom.

- Kutzback's equation

$F = 3(L - 1) - 2j - h$

Here, L = Number of link

j = Number of binary joint

h = Number of higher joint

- Grubler's Equation

For those machanism which have single degree of freedom and zero higher pair.

$3l - 2j - 4 = 0$

Here,

l = Number of links

j = Number of binary joints

- Degree of Freedom

F = 0 (Frame)

F < 0 (redundant frame), indeterminate streucture

F > 0 (constrained/unconstrained frame)

Point to remember:

- All machanism have minimum 4 number of link.

Kinematic Chain

When all the links are connected in such a way that first link is jointed with the last link, then the structure formed is known as closed chain. The closed chain will be a kinematic chain when the relative motion between the link is either completely constrained or successfully constrained.

- Condition for a kinematic chain

  There are two conditions and both the conditions are equivalent.

- Relation between the number of links and number of pair.

  $l = 2P - 4$

  where, l = Number of link

  P = Number of pair

- Relation between number of binary joints and number of links

  $2J = 3l - 4$

  where, J = Number of binary joint.

  l = Number of link

  (i) If L.H.S. > R.H.S.

  Locked chain or frame or structure

  (ii) If L.H.S. = R.H.S

  Kinematic chain (constrained chain)

  (iii) If L.H.S. < R.H.S

  unconstrained chain

  Redundant chain

  It does not allow any motion of a link relative to other.

  Frame/Structure

  If one of the link of redundant chain is fixed. It is known as structure or a locked system.

- Points to remember:

  No relative motion

  Capable of transmitting force only.

  Power/energy can not be transmitted.

  Degree of freedom of *structure* is zero.

  Degree of freedom of super structure is less than zero.

  Simple Machanism

  All the machanism having 4-links are simple machanism and the mechanism having more than 4 links are compound mechanism.

  There are there different simple machanisms.

- Four bar machanism/Quadric cycle chain.

  It consists of four link and four turnining pair.

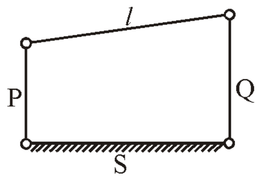
- Slider crank machanism

  It consists four link, three turning pair and one sliding pair.

- Double slider crank machanism.

  It consists four link, two turning pair and two sliding pair.

Four-Bar Machanism



➢ Grashof's law

$S + l \leq P + Q$

Here,

S = Shortest link length

l = Longest link length

P,Q = Adjacent link length to shortest link.

Case-I:

If $S + l < P + Q$

➢ S is fixed – Double crank mechanism.

➢ P or Q fixed – Crank rocker mechanism

➢ l is fixed – Rocker-rocker mechanism

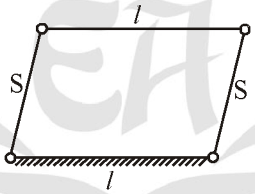
Case-II:

If $S + l = P + Q$

➢ All link have different length then same as case-1.

➢ Parallelogram linkage-crank-crank mechanism.

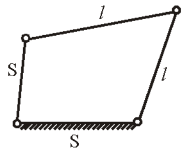
i.e. $S = P$, $l = Q$



S is fixed – Double crank mechanism.

l is fixed– Double crank mechanism.

Deltoid linkage



(a) S is fixed – Crank-crank mechanism

(b) l is fixed – Crank-rocker mechanis.

Case-III:

$s + l > P + Q$

Grashof's law is not satisfied and it will give non grashof's rocker-rocker mechanism.

Inversion of 4-Bar Mechanism

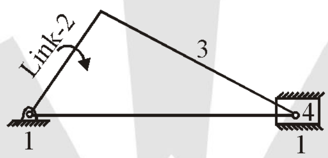
Coupling rod of locomotive	–	Crank-crank mechanism
Beam engine	–	Crank-rocker mechanism
Watt's indicator	–	Rocker-rocker mechanism

Points to remember:

Approximate straight line mechanism are watts indicator, modified Scott-russel mechanism (Grass Hopper mechanism). The tchebicheff streight line mechainsm.

Exact straight line mechanism are Peculiar mechanism, Hart mechanism, Scott-russel mechanism.

Inversion of Slider-Crank Mechanism



First inversion-link 1 is fixed.

Reciprocating engine/compressor.

Second inversion-link 2 is fixed (Crank)

Whitworth quick return mechanism, rotary (redial) engine.

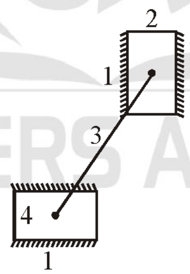
Third inversion-link 3 is fixed (Connecting Rod)

Crank and slotted lever mechanism, oscillating cylinder mechanism.

Fourth inversion-link 4 is fixed (Slider)

Hand pump, bull engine.

Inversion of Double Slider Crank Mechanism



First inversion-link 1 is fixed

Elliptical trammel.

Second inversion-slider 2 is fixed.

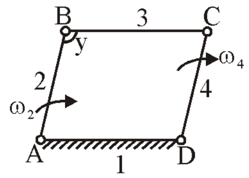
Scotch yoke mechanism.

Third inversion-link 3 is fixed

Oldham coupling.

Mechanical Advantage

$$M.A. = \frac{\text{Output force / torque}}{\text{Input force / torque}}$$



Power input = Power output

$$T_2\omega_2 = T_4\omega_4$$

$$M.A. = \frac{T_4}{T_2} = \frac{\omega_2}{\omega_4}$$

Points to remember:

If γ is equal to 0° or 180°. ω_4 become zero thus mechanical advantage will be infinity.

Extreme position of linkage is knows as toggle position.

2. Velocity Analysis

Velocity in machines can be dtermined by either analytically or graphically. This chapter deals with graphical analysis.

Velocity Analysis

Let

V_{ao} = Velocity of A relative to O

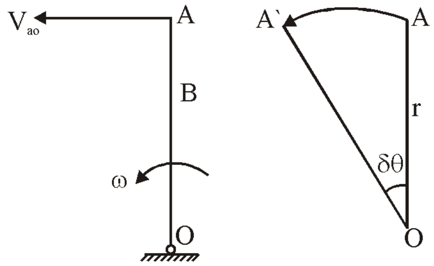
V_{ba} = Velocity of B relative to A

V_{bo} = Velocity of B relative to O

$V_{bo} = V_{ba} + V_{ao}$

Velocity of A relative to O = $\frac{\text{Arc AA`}}{\delta t}$

$$V_{ao} = \frac{r.\delta\theta}{\delta t} = r\frac{d\theta}{dt} = r\omega$$



as δt → O, AA` will be perpendicular to OA, Thus the velocity of A is ωr and is perpendicualr to OA.

Points to remember:

The velocity of any point relative to any other point on a fixed link is always zero.

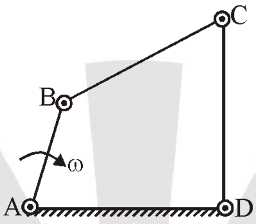
The velocity of an intermediate point on any of links can be found easily by dividing the corresponding velocity vector in the same ratio as the point divides link.

The angular velocity of a link about one extremity is the same as the angular velocity about the other.

Velocity of Rubbing

The velocity of rubbing of the two surface will depend upon the angular velocity of a link, relative to the other.

If pin at A



Pin at A joins links AD and AB. AD being fixed the velocity of rubbing will depend only upon angular velocity of AB.

∴ Velocity of rubbing

$$= r_a \, \omega_{AB}$$

Here, r_a = Radius of pin A

Pin at D

Velocity of rubbing = $r_d \cdot \omega_{cd}$

Pin at B

Both link AB and BC is moving

$$\omega_{ab} = \omega = \text{clockwise}$$

$$\omega_{bc} = \omega = \text{anticlockwise}$$

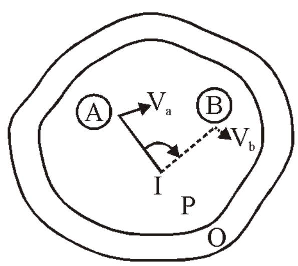
∴ Velocity of rubbing = $r_b\,(\omega_{ab} + \omega_{bc})$

Pin at C

Velocity of rubbing = $r_c\,(\omega_{bc} + \omega_{dc})$

Theory of Instantaneous Centre

Instantaneous centre of rotation or vitrual centre.



Let a plane body 'P' having non-linear motion relative to another plane body Q. At any instant, the linear velocities of the point 'A' and 'B' on the body 'P' are 'V_a' and 'V_b' respectively.

If a line is drawn perpendicular to the direction of V_a at 'A', the body can be imagined to rotate about some point on this line. Similarly for point B. If the intersection of the two lines is at 'l', the body 'P' will be rotating about *l* at the instant.

This point 'l' is known as instantaneous centre of velocity.

Point to remember:

If the direction V_a and V_b are parallel to the l-centre of body lies at infinity.

Centrode:

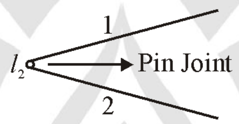
As we know, in general the position of instantaneous centre changes throughout the whole motion. The locus of all these instantaneous centre for a particular link is known as 'Centrode'. It is a *curve*.

Axode:

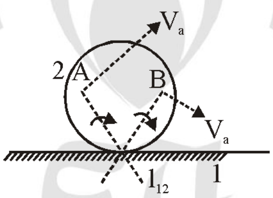
The line passing through instantaneous centre and perpendicular to the plane of motion is known as instantaneous axis. The locus of instantaneous axis for a link during the whole motion is known as 'Axode'. It is a *surface*.

Instantaneous Centre in Different Situation

If two links are attached with a turning pair, the instantaneous centre of such a pair will be at *pin joint*.

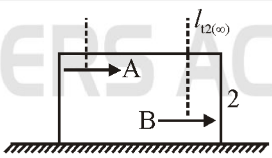


Rolling of a sphere on a plane.



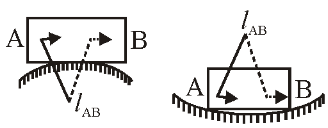
Instantaneous centre will be at point of contant.

Object sliding on a plane surface.



The location of l_{12} will be at infinity but in a direction perpendicular to the *sliding surface.*

Object sliding on a *curved* surface.



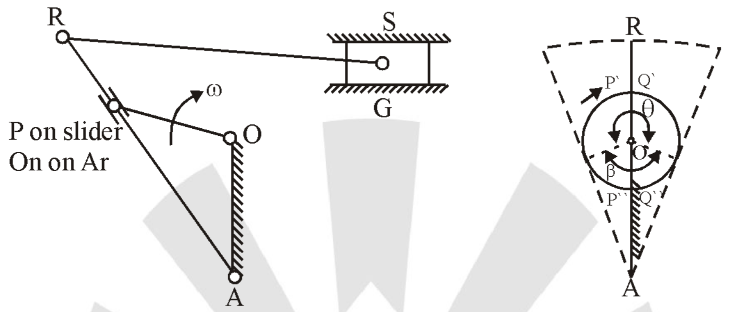
Instantaneous centre will be at the *centre* of curvature of curved surface.

Number of Instantaneous Centre

$$l = \frac{n(n-1)}{2}$$

Here, n = Number of link

Crank and Slotted Lever Mechanism



Let,

r = length of crank (= OP)

l = Length of slotted lever (=AR)

C = Distance between fixed centres (=AO)

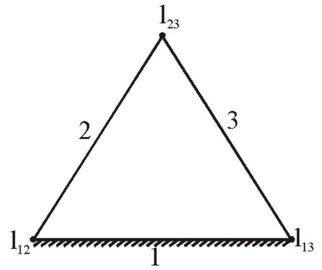
ω = angular velocity of crank

Thus during cutting stroke

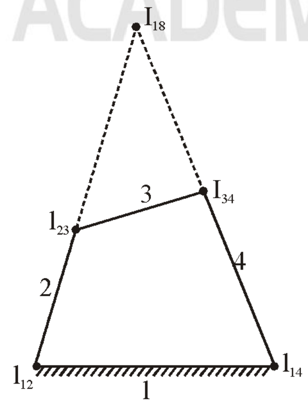
$$\frac{(V_s)_{max}(\text{cutting})}{(V_s)_{max}(\text{return})} = \frac{c-r}{c+r}; \quad \frac{\text{Time of cutting}}{\text{Time of return}} = \frac{\theta}{\beta}$$

Kennedy's Theorem

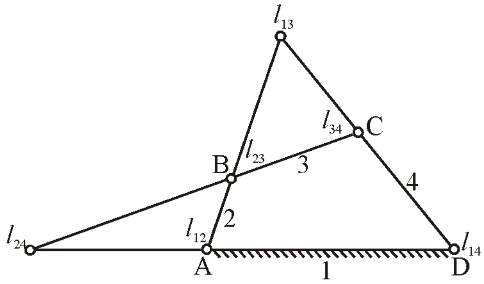
For the three bodies having the contiuous relative motion there all instaneous centres lies on the same line.



No relative motion



Angular Velocity Ratio Theorem



It is used to find angular velocity of a link if angular velocity of another link is known.

The angular velocity ratio of two links relative to third link is inversely proportional to the distances of their common l-centre from their respective centre of rotation.

$$\frac{\omega_4}{\omega_2} = \frac{l_{24} - l_{12}}{l_{24} - l_{14}}$$

Angular velocity ratio is positive when the common instantanteous center falls outside the other two centers & negative when it falls between them.

3. Acceleration Analysis

The rate of change of velocity with respect to time is known as acceleration and it acts in the direction of the change in velocity.

Tangential and Centripetal Acceleration

The rate of change of velocity in the tangential direction of the motion is known as tangential acceleration.

$$a_t = \frac{dv}{dt}$$

The rate of change of velocity towards the centre of rotation is known as centropetal or radial acceleration.

$$a_c = \frac{v^2}{r}$$

Points to remember:

The tangential component of acceleration occurs due to the angular accelration of link.

the acceleration of intermediate points on the links can be obtained by dividing the acceleration vectors in the same ratio as the points divide the links.

Coriolis Acceleration Component

Coriolis Acceleration Component

Let,

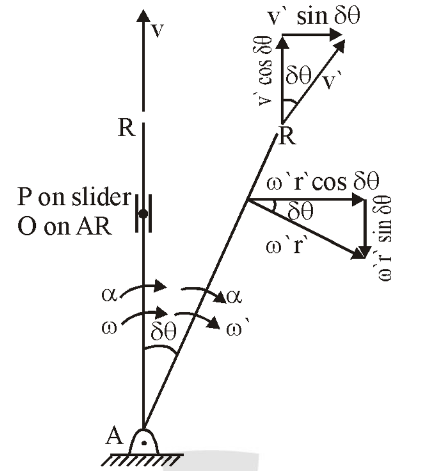
ω = Angular velocity of the link

α = Angular acceleration of the link

v = Linear velocity of the slide of the link

f = Linear acceleration of the slider on the link

r = Radial distance of point P on eht slider.



Acceleration of P along AR =

Acceleration of slider – Centrifugal acceleration

$= f - \omega^2 r$

Acceleration of P perpendicular to AR

= 2 wV + Tangential acceleration

$= 2\,\omega V + r\alpha.$

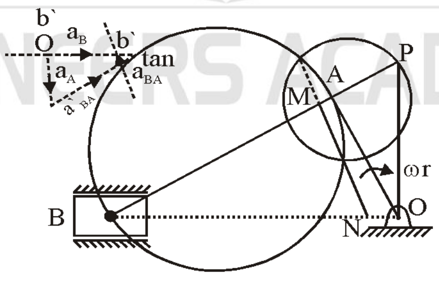
Points to Remember:

The component 2 ωv is known as the Coriolis acceleration component.

The direction of the coriolis acceleration component is obtained by rotating the radial velocity vector 'v' through 90° in the direction of rotation of the link.

Klein's Construction

In kleni's construction, the velocity and the acceleration diagrams are made on the configuration diagram itself. The line that represents the crank in the configuration diagram also represents the velocity and acceleration of its moving end in the velocity and acceleration diagram respectively.



$$\frac{a_B}{ON} = \frac{a`B_A}{AM} = \frac{a_A}{OA} = \frac{a_{BA}^{tan}}{NM} = \omega^2$$

OAP – Velocity diagram

OAMN – Acceleration diagram.