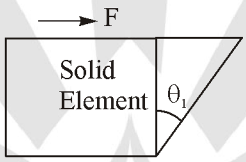
# INTRODUCTION

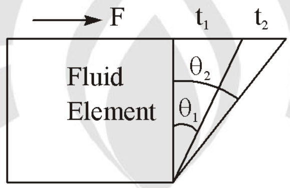
## THEORY

## 1.1 Fluid

The substances which continuously deform under the application of a tangential or shear stress no matter how small is the value of tangential force is known as fluid.




***θ is the deformation at time $t_1$ and $t_2$ if force(F) is constant***




***$\theta_1$ at time $t_1$; $\theta_2$ at time $t_2$ if $t_2 > t_1$ then $\theta_2 > \theta_1$ for fluid element***

where,
$\theta_1$ = Deformation at time $t_1$
$\theta_2$ = Deformation at time $t_2$

## 1.2 Properties of Fluid

### *1.2.1 Density or Mass Density*

Mass density is define as the mass of the system corresponding to its volume.



$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{\forall}$$

$$\text{Unit} = \left(\frac{kg}{m^3}\right)$$

$$\text{Dimensional formula} = \left[M^1L^{-3}T^0\right]$$

For ideal gases

$$P\forall = mRT$$

$$P = \frac{m}{\forall}RT$$

$$P = \rho RT$$

where, R is the gas constant

$$R = 0.287\,\frac{kJ}{kg.k} \text{ for air.}$$

***Note:***

a) Density is absolute quantity.

b) Density of gas is dependent on the temperature and pressure of the system.

### *1.2.2 Specific Gravity (S)*

$$S = \frac{\text{density of substance}}{\text{density of standard fluid}}$$

***Note:***

a) For liquid standard fluid is water (1000 kg/$m^3$)

b) For gases standard fluid is either air or hydrogen gas.

c) Specific gravity of Hg is 13.6

### *1.2.3 Relative Density (R.D.)*

$$\text{Relative density (R.D.)} = \frac{\text{density of one fluid}}{\text{density of another fluid}}$$

$$R.D. = \frac{\rho_1}{\rho_2}$$

### *1.2.4 Specific Weight or Weight Density (w or $\gamma$):*

It is define as the ratio of weight of system to the volume of the system.

$$w = \frac{\text{weight}}{\text{volume}} = \frac{\rho g \forall}{\forall} = \rho g$$

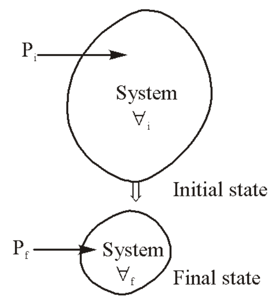
$$\text{Unit} \quad \left(\frac{kg}{m^3} \times \frac{m}{sec^2}\right) = \left(\frac{kg}{m^2.sec^2}\right)$$

$$\text{Dimensional formula} = \left[M^1 L^{-2} T^{-2}\right]$$

***Note:***

Specific weight is the relative quantity and it depends on the pressure, temperature and location.

### *1.2.5 Bulk Modulus and Compressibility*




If original pressure is P in a liquid mass and its volume is $\forall$ and increase in pressure dP causes change in volume d$\forall$ then

$$dP = P_f - P_i$$

$$d\forall = \forall_f - \forall_i = -(\forall_i - \forall_f)$$

$$\text{Bulk modulus of elasticity (K)} = -\frac{dP}{(d\forall/\forall)} \qquad \left[\text{analogous to } \frac{\text{stress}}{\text{strain}} = \text{Young's modulus of elasticity}\right]$$

Unit of K will same as pressure (Pa, N/$m^2$)

$$\boxed{K = -\frac{dP}{\left(\frac{d\forall}{\forall}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)}}$$

$$\rho\forall = \text{mass} = \text{constant}$$

$$\rho d\forall + \forall d\rho = 0$$

$$\Rightarrow \quad \frac{d\rho}{\rho} = -\frac{dV}{\forall}$$

$$\boxed{\text{Compressibility} = \frac{1}{K}} = \frac{1}{\rho}\frac{d\rho}{dP}$$

***Note :*** If density does not change with pressure i.e., $\frac{d\rho}{dP} = 0$ (fluid is incompressible with respect to pressure).

### *1.2.6 Isothermal Bulk Modulus ($K_T$)*

For ideal gas $\quad P = \rho RT$

$$R = 0.287\frac{kJ}{kg - k}$$

$$\Rightarrow \quad \left(\frac{dP}{d\rho}\right)_{T=\text{Constant}} = RT$$

$$\Rightarrow \quad K_T = \rho\frac{dP}{d\rho} = \rho \cdot RT = P$$

$\Rightarrow$ for isothermal condition in an ideal gas,

$$K_T = P$$

i.e, $\boxed{\text{Isothermal bulk modulus = pressure}}$

### *1.2.7 Adiabatic Bulk Modulus ($K_A$)*

For adiabatic condition $\quad P\forall^{\gamma} = \text{constant}$

where, $\quad \gamma = \text{adiabatic index}$

$$= \frac{Cp}{Cv} = \frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}$$

$$\Rightarrow \quad P\left(\frac{m}{\rho}\right)^{\gamma} = \text{constant}$$

$$\frac{P}{\rho^{\gamma}} = \text{constant (because mass 'm' is constant)}$$

$$\Rightarrow \quad \boxed{P = C\rho^{\gamma}}$$

$$\frac{dP}{d\rho} = \rho C\rho^{(\gamma-1)}$$

$$K_A = \rho\frac{dP}{d\rho} = \gamma C\rho^{\gamma} = \gamma P$$

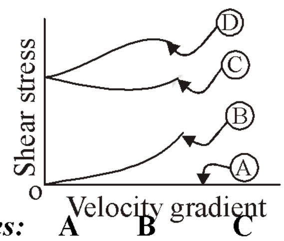
$$\Rightarrow \quad \boxed{\text{Adiabatic bulk modulus} = \gamma P = \text{Adiabatic index} \times \text{Pressure}}$$

# PRACTICE SHEET

## OBJECTIVE QUESTIONS

**1.** Match List-I (Curves labelled A, B, C and D in figure) with List-II (Type of fluid) and select the correct answer using the codes given below the lists:

**List-I**




**List-II**

1. Ideal plastic
2. Ideal
3. Non-Newtonian
4. Rheopectic
5. Thixotropic

***Codes:***

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 2 | 3 | 1 | 5 |
| (b) | 3 | 2 | 5 | 1 |
| (c) | 4 | 2 | 5 | 1 |
| (d) | 2 | 3 | 5 | 1 |

**2.** If bulk modulus of water is $2 \times 10^8$ kgf/cm$^2$ ($19.62 \times 10^8$ N/m$^2$), then water hammer wave celerity through a rigid pipe line will be

(a) 600 m/s (b) 800 m/s

(c) 1200 m/s (d) 1400 m/s

**3.** Surface tension is due to

(a) Cohesion and adhesion

(b) cohesion only

(c) Adhesion only

(d) None of the above

**4.** Continuum approach in fluid mechanics is valid when

(a) The compressibility is very high

(b) The viscosity is low

(c) The mean free path of the molecule is much smaller compared to the characteristic dimension

(d) M >> 1, where M is the Mach number

**5.** A fluid is said to be Newtonian fluid when the shear stress is

(a) directly proportional to the velocity gradient

(b) inversely proportional to the velocity gradient

(c) independent of the velocity gradient

(d) none of the above

**6.** A fluid is one which can be defined as a substance that:

(a) has same shear stress at all points

(b) can deform indefinitely under the action of the smallest shear force

(c) has the small shear stress in all directions

(d) is practically incompressible

**7.** The dimension of surface tension is:

(a) $ML^{-1}$ (b) $L^2V^{-1}$

(c) $ML^{-1}\ T^{-1}$ (d) $MT^{-2}$

**8.** A dimensionless combination of pressure drop ΔP, dynamic viscosity μ, velocity V and length L is

(a) $\dfrac{\Delta P}{V^2.L}$ (b) $\dfrac{VL}{\mu}$

(c) $\dfrac{\Delta P}{\mu.VL}$ (d) $\dfrac{\Delta P.L}{\mu.V}$

**9.** Shear stress in the Newtonian fluid is proportional to

(a) pressure

(b) strain

(c) strain rate

(d) the inverse of the viscosity

**11.** With increase of temperature, viscosity of a fluid

(a) Does not change

(b) Always increases

(c) Always decreases

(d) Increases, if the fluid is a gas and decreases, if it is a liquid

**12.** The unit of dynamic viscosity of a fluid is

(a) $m^2/s$ (b) $\dfrac{N \cdot s}{m^2}$

(b) $\dfrac{Pa \cdot s}{m^2}$ (d) $\dfrac{kg \cdot s^2}{m^2}$

**13.** The unit of surface tension is:

(a) $N/m^2$ (b) J/m

(c) $J/m^2$ (d) W/m

**14.** If 'P' is the gauge pressure within a spherical droplet, then gauge pressure within a bubble of the same fluid and of same size will be:

(a) $\frac{P}{4}$ (b) $\frac{P}{2}$

(c) P (d) 2P

**15.** Kinematic viscosity of air at 20° C is given to be $1.6 \times 10^{-5}$ m²/s. Its kinematic viscosity at 70° C will be varying approximately:

(a) $2.2 \times 10^{-5}$ m²/s (b) $1.6 \times 10^{-5}$ m²/s

(c) $1.2 \times 10^{-5}$ m²/s (d) $3.2 \times 10^{-5}$ m²/s

**16.** Match List-I (Fluid properties) with List-II (Related terms) and select the correct answer using the codes given below the lists:

| List-I | List-II |
|---|---|
| **A.** Capillarity | **1.** Cavitation |
| **B.** Vapour pressure | **2.** Density of water |
| **C.** Viscosity | **3.** Shear forces |
| **D.** Specific gravity | **4.** Surface tension |

***Codes:***

| | A | B | C | D |
|---|---|---|---|---|
| (a) | 1 | 4 | 2 | 3 |
| (b) | 1 | 4 | 3 | 2 |
| (c) | 4 | 1 | 2 | 3 |
| (d) | 4 | 1 | 3 | 2 |

**17.** Which one of the following is the bulk modulus K of a fluid? (Symbols have the usual meaning)

(a) $\rho\frac{dp}{d\rho}$ (b) $\frac{dp}{\rho d\rho}$

(c) $\rho\frac{d\rho}{dp}$ (d) $\frac{dp}{\rho dp}$

**18.** Which of the following forces act on a fluid at rest?

**1.** Gravity force

**2.** Hydrostatic force

**3.** Surface tension

**4.** Viscous force

Select the correct answer using the codes given below:

(a) 1, 2, 3 and 4 (b) 1, 2 and 3

(c) 2 and 4 (d) 1, 3 and 4

**19.** Surface tension is due to

(a) viscous forces

(b) cohesion

(c) adhesion

(d) the difference between adhesive and cohesive forces

**20.** Newton's law of viscosity depends upon the

(a) stress and strain in a fluid

(b) shear stress, pressure and velocity

(c) shear stress and rate of strain

(d) viscosity and shear stress

**21.** If the surface tension of water-air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter will be

(a) 0.146 N/m²

(b) 73 N/m²

(c) 146 N/m²

(d) 292 N/m²

**24.** If the volume of a liquid decreases by 0.2 percent for an increase of pressure from 6.867 MN/ m² to 15.696 MN/m², what is the value of the bulk modulus of the liquid ?

(a) 4418 MPa (b) 441.8 MPa

(c) 4.418 MPa (d) 44180. MPa

**25.** If a certain liquid has a viscosity of 0.048 poise and kinematic viscosity $3.50 \times 10^{-2}$ stokes, what is its specific gravity?

(a) 1.23142 (b) 1.001

(c) 1.37142 (d) 1.17353

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## ANSWERS AND EXPLANATIONS

1. ***Ans. (a)***

Horizontal line representing zero shear stress for any velocity gradient is the condition for ideal fluid.

The curve B represents dilatant fluid.

$$\boxed{\tau = \mu\left(\frac{du}{dy}\right)^n \; n > 1}$$

For pseudoplastic fluid, $\boxed{\tau = \mu\left(\frac{du}{dy}\right)^n \; n < 1}$

For thixotropic fluid, $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)^n \; n < 1}$

For rheopectic fluid, $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)^n \; n > 1}$

For plastic fluid, $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right) \text{i.e., } n = 1}$

2. ***Ans. (d)***

$$C = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{19.62 \text{ x } 10^8}{1000}} = 1400 \text{ m/s}$$

K = bulk modulus of elasticity

ρ = mass density of water

3. ***Ans. (b)***

Surface tension is due to cohesive force only.

4. ***Ans. (c)***

Continuum approach means properties uniform through out the medium. This condition is valid only when mean free path (λ) is much lower as compare to characteristic length ($L_C$).

5. ***Ans. (a)***

For newtonian fluid

$$\boxed{\tau \alpha \frac{du}{dy} \alpha \frac{d\theta}{dt}}$$

6. ***Ans. (b)***

Fluid is substance which deform continuously under the action of small shear or tangential force.

7. ***Ans. (d)***

Surface tension $= \frac{F}{\ell}$

$$\boxed{= \left(\frac{N}{m}\right) = \frac{kg \cdot m/s^2}{m} = [M^1 L^0 T^{-2}]}$$

8. ***Ans. (d)***

$$\Delta P = N/m^2 = kg.\frac{m}{sec^2}/m^2 = M^1.L^{-1}.T^{-2}$$

$$V = m/s = L^1T^{-1}$$

$$L = M = L^1$$

$$\mu = \frac{N-S}{m^2} = kg/m-s = M^1.L^{-1}T^{-1}$$

$$\mu = \frac{N-S}{m^2} = kg/m-s = m^1.L^{-1}T^{-1}$$

Check:

(a) $\frac{\Delta P}{V^2 \cdot L} = \frac{M^1L^{-1}T^{-2}}{L^2T^{-2}.L^1} = M^1L^{-4}$

(b) $\frac{V.L}{\mu} = \frac{L^1T^{-1}L^1}{M^1L^{-1}T^{-1}} = M^{-1}L^{-4}$

(c) $\frac{\Delta P}{\mu.V.L.} = \frac{M^1L^{-1}T^{-2}}{M^1L^{-1}T^{-1}.L^1T^{-1}L^1} = L^{-2}$

(d) $\frac{\Delta P.L}{\mu.V} = \frac{M^1L^{-1}T^{-2}.L^1}{M^1L^{-1}T^{-1}.L^1T^{-1}} = M^0L^0T^0$

9. ***Ans. (c)***

$$\tau \alpha \frac{du}{dy} \alpha \frac{d\theta}{dt}$$

where, θ is angular strain

$\frac{du}{dy}$ has it's unit is $s^{-1}$ i.e., rate of strain.

11. ***Ans. (d)***

Viscosity is due to cohesion in liquids. As a temperature increases cohesion of liquids decrease. Hence as $T(\uparrow)\mu$ of liquids decrease. In case of gases, viscosity depends on molecular momentum exchange. As temperature, increases, molecular activity of gases increase and hence resistance to flow increase. Hence as temperature increases viscosity of gases increases.

**12.** ***Ans. (b)***

$$\tau = \mu\frac{du}{dy}$$

$$\frac{N}{m^2} = \mu\frac{m/s}{m}$$

$$\boxed{\mu = \left(\frac{N-s}{m^2}\right)}$$

**13.** ***Ans. (c)***

Surface tension $= \dfrac{F}{\ell} = \dfrac{N}{m}\times\dfrac{m}{m} = J/m^2$

**14.** ***Ans. (d)***

$\Delta P = \dfrac{4\sigma}{D}$ in case of drop.

$\Delta P = \dfrac{8\sigma}{D}$ in case of bubble

and hence $(\Delta P)_{bubble} = 2\ (\Delta P)_{drop}$

**15.** ***Ans. (a)***

**For Air**

$$\mu \propto \sqrt{T}\ \ \&\ \ \rho \propto 1/T$$

$$v = \frac{\mu}{\rho}$$

$$v \propto \frac{\sqrt{T}}{1/T}$$

$$v \propto T^{3/2}$$

**16.** ***Ans. (d)***

***Viscosity :*** It is a measure of resistance of a fluid which is being deformed by either shear stress or tensile stress.

***Specific gravity :*** It is the ratio of density of fluid to the density of standard fluid.

***Capillarity :*** It is the ability of liquid to flow against gravity combination of surface tension and adhesion act to lift the liquid.

**17.** ***Ans. (a)***

Bulk modulus $K = -\dfrac{dp}{d\forall/\forall}$

and $\forall = \dfrac{m}{\rho}$

$\therefore\ d\forall = -\dfrac{d\rho}{\rho^2}$

$$K = -\frac{dp}{\dfrac{-d\rho/\rho^2}{1/\rho}}$$

$$K = \frac{\rho dp}{d\rho}$$

**18.** ***Ans. (b)***

A fluid at rest there can be no shear force (i.e. viscous force). The only forces acting on the free body are the normal pressure forces, exerted by the surrounding fluid on the plane surface and the weight of the element.

**19.** ***Ans. (b)***

Surface tension is due to cohesion between liquid particles at the surface, where as capillarity is due to both cohesion and adhesion.

The property of cohesion enables a liquid to resist tensile stress, while adhesion enables it to stick to another body.

**20.** ***Ans. (c)***

Newton's law of viscosity

$\tau = \mu\dfrac{du}{dy}$ Where $\tau$ = shear stress

$\dfrac{du}{dy}$ = Rate of strain

**21.** ***Ans. (d)***

Pressure intensity inside a droplet

$$\Delta P = \frac{4\sigma}{d} = \frac{4\times 0.073}{10^{-3}} N/m^2 = 292\ N/m^2$$

**24.** ***Ans. (a)***

$$\frac{\Delta V}{V} = -0.002$$

$$P_1 = 6.867\times 10^6\ \frac{N}{m^2}$$

$$P_2 = 15.696\times 10^6\ \frac{N}{m^2}$$

hence bulk modulus is

$$K = \frac{-dp}{d\forall | \forall} = \frac{\left(15.696 \times 10^6 - 6.86 \times 10^6\right)}{0.002} \frac{N}{m^2}$$

$$= 4.418 \times 10^9 \frac{N}{m^2}$$

$$= 4418 \, MN / m^2$$

**25.** ***Ans. (c)***

Given that

$$\mu(\text{dynamic cis cosity}) = 0.048 \times 10^{-1} \frac{N-s}{m^2}$$

$\upsilon$ = 3.50 × $10^{-2}$ stroke

= 3.50 × $10^{-6}$ m $^2$/sec

we know that

$$\frac{\mu}{\rho} = \upsilon$$

$$\upsilon = \left(\frac{\mu}{\rho}\right) = \frac{0.048 \times 10^{-1} \frac{N-s}{m^2}}{3.50 \times 10^{-6} m^2 / sec}$$

$$\rho = 1.31742 \text{ kg} / m^3$$

$$S = \frac{\rho}{1000}$$

S = 1.31742

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# 2 PRESSURE & ITS MEASUREMENT

## THEORY

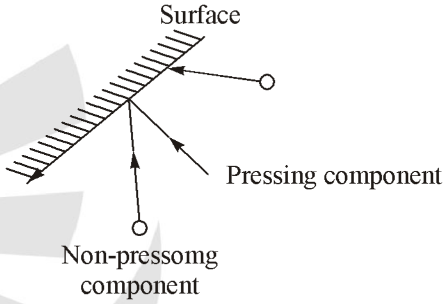
## 2.1 Pressure

Pressure is define as a normal compressive external force acting on a per unit cross section area. Existence of mass is the existence of pressure.

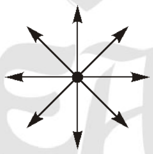
***Mathematically,***

$$P = \frac{F}{A}$$

F = external normal force
or external thrust
or pressure force




From the figure below we can say that pressure is a scalar quantity:

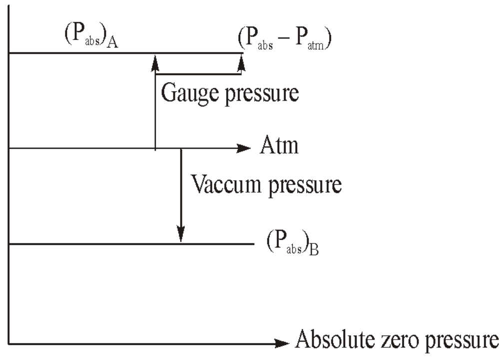


From the figure pressure have infinite direction, so that pressure is a scalar quantity.

### *2.1.1 Different Types of Pressure*

$(P_{abs})_A = P_{atm} + (P_{gauge})_A$ (when above the $P_{atm}$)

$(P_{abs})_B = P_{atm} - (P_{vaccum})_B$ (when below the $P_{atm}$)



**(a) Atmospheric Pressure :**

It is the pressure exerted by environmental mass on the body.

**(b) Absolute Pressure :**

It is the net pressure exerted on a system with respect to zero pressure level.

**(c) Gauge Pressure :**

It is the pressure above the atmospheric pressure.

**(d) Vacuum Pressure**

It is the pressure below the atmospheric pressure.

–ve gauge pressure ⇒ vacuum pressure.

–ve absolute pressure will never exist. (It may become zero).

P.S.I. ⇒ Pound force per square inch

$$1\text{ P.S.I.} = \frac{\ell\text{bf}}{(\text{inch})^2} = \frac{0.453}{(2.53)^2}\frac{\text{kgf}}{\text{cm}^2} = \frac{0.453}{(2.53)^2}\times 9.81\times 10^4\ \text{Pa}$$

$$\boxed{1\text{P.S.I.} = 6942.66\text{ Pa}}$$

$\because$

$$1\text{ P.S.I.} = 6942.66\text{ Pa}$$
$$1\text{ atm} = 101325\text{ Pa}$$
$$\boxed{1\text{atm} = 14.6\text{ P.S.I.}}$$

### *2.1.2 Unit of Pressure*

Pressure is defined as normal force exerted by a fluid per unit area. The various units in which pressure is measured are given as under.

1 pascal = $1\text{N/m}^2$

1 MPa = 1 $\text{N/mm}^2$

1 bar = $10^5$ pascal = 0.1 $\text{N/mm}^2$

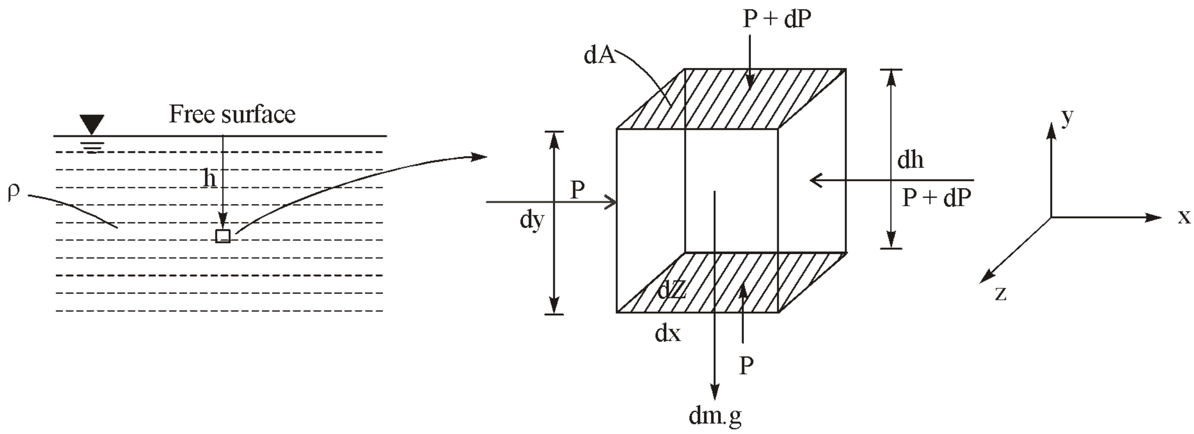
1 Atm = 101.325 kPa = 0.101325 MPa

1 Atm = 1.01325 bar

1 $\text{kgf/cm}^2$ = 9.807 $\text{N/cm}^2$ = 0.9807 bar = 0.9679 Atm.

1 Atm = 14.396 PSi (Pound per sq. inch).

## 2.2 Pressure at a Certain Point in Static Fluid or Hydrostatic Pressure at Certain Point or Pascal's Law



Force balancing in vertical direction

$$PdA - (P + dP)dA - dmg = 0$$

$$dPdA = -dm.g \qquad \because dm = \rho d\forall$$

$$dPdA = -\rho dA.dh.g \qquad dm = \rho dA.dh$$

$$dP = -\rho g\, dh$$

Pressure gradient in downward direction $\boxed{\dfrac{dP}{dh} = -\rho g}$

$$\int_{P_{atm}}^{P} dP = \rho g \int_{0}^{-h} -dh$$

$$\boxed{P - P_{atm} = \rho g h}$$

Force balancing in X-direction

$$PdA - (P + dP)dA = 0$$

$$dP = 0$$

Similarly in Y-direction

$$dP = 0$$

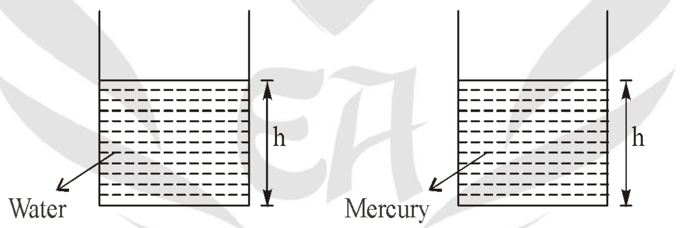
*Note:* (a) It means pressure will vary only in vertical direction.

(b) The mohr circle for a static fluid is a point.

***2.2.1 Statement :***

According to Pascal's Law pressure at any given location is same in all direction when fluid is at rest condition

***2.2.2 Pressure in Terms of Height of Column***




h ⇒ vertical height

For water

$$1\text{ atm} = \rho_w.g.h$$

$$101325\text{ Pa} = 1000 \times 9.81 \times h$$

$$\boxed{h = 10.3\text{m}}$$

For Hg

$$1\text{ atm} = \rho_{Hg}.g.h$$

$$101325\text{ Pa} = 13.6 \times 10^3 \times 9.81 \times h$$

$$\boxed{h = 760\text{ mm}}$$

## 2.3 Pressure Measurement

Different instruments for the measurements of pressure

Conventional
- Barometer
- Piezometer
- Manometers

Modern
- Bourden Gauges
- Strain gauge transducer
- Piezo electronic transducer

# BUOYANCY AND FLOATATION

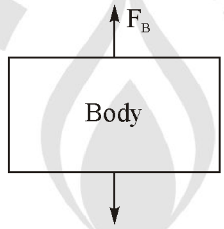
## THEORY

### 3.1 Concept of Buoyancy and Floating (Archimedes Principle)

When a body is submerged or emerged (fully or partially) inside the fluid, then fluid will exert a force on the body. The net hydrostatic force act on a body by the fluid in the vertical upward direction is known as "up-thrust" or Buoyancy force and the magnitude of this force is equal to the weight of the displaced fluid.

***In general condition***

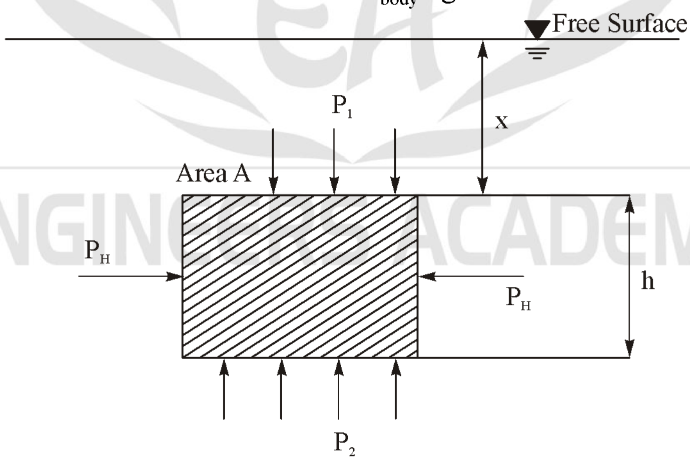
For analysis, draw FBD of body




Weight = $(m_{body}) \times g$

Upward thrust = $F_B$

Downward thrust = $m_{body} \cdot g$




$$F_1 = P_1 A \qquad P_1 = \rho g x$$

$$F_1 = (\rho g x) A \;;$$

$$F_2 = P_2 A \qquad P_2 = \rho g(x + h)$$

$$F_2 = \rho g (x + h) A$$

$$F_2 - F_1 = \text{Net upward}$$

Force on body due to water on bottom surface.

$$= \rho g\, hA$$

$$= \rho g \times \text{volume}$$

= weight of displaced fluid

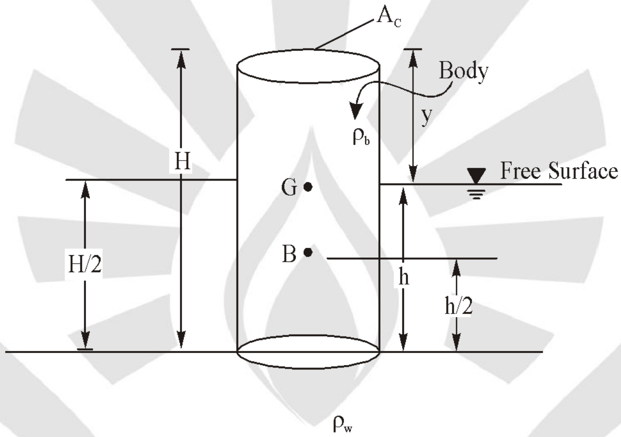
Buoyant force = weight of liquid displaced

*Note :* Point of application of this force is at the C.G. of displaced liquid. C.G. of displaced liquid is called centre of Buoyancy.

### *3.1.1 Concept of Floatation*

A body will float in a liquid, if the weight of body is equal to the weight of liquid displaced by the submerged portion of body.

Assume that the body have density $\rho_b$ in floating of water ($\rho_w$).




The shape of body is cylindrical having height H and cross section area $A_C$.

***For equilibrium condition :***

downward force = upward force

$$A_C H \rho_b . g = A_C h \rho_w g$$

$$H\rho_b = h\rho_w$$

$$H\rho_b = h\rho_w$$

$$\boxed{h = \frac{H\rho_b}{\rho_w}}$$

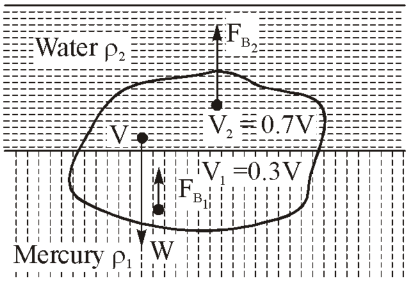
**Example 1 :** A metallic body Floats at the interface of mercury (S = 13.6) and water such that 30% of its volume is submerged in mercury and 70% in water Estimate the Density of the metal.

***Solution :*** The Buoyancy Force due to mercury

$$F_{B_1} = (\rho_1 g V_1)$$

$$= (13.6 \times 1000 \times g \times 0.3\ V)$$

$$= 4080\ gV \qquad \text{... (i)}$$




The Buoyancy Force due to water = $(\rho_2 \text{ g } V_1)$

$$= (1000 \text{ g} \times 0.7 \text{ V}) = 700 \text{ gV} \quad ...(ii)$$

From the Archimedes principle of Floatation

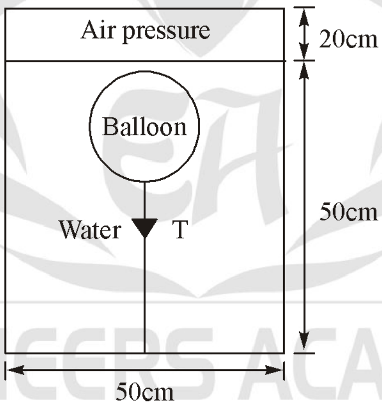
$$\Rightarrow \quad W = \left(F_{B_1} + F_{B_2}\right)$$

$$\Rightarrow \quad \rho g V = \left(4080gV + 700gV\right)$$

$$\Rightarrow \quad \rho = \frac{4780\,gV}{gV}$$

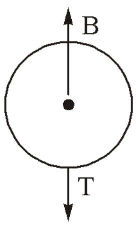
$$\boxed{\rho = 4780\,\text{kg/m}^3}$$

**Example 2 :** Assuming weight of Balloon to be negligible, Find out the percentage change in tension in the string when pressure is changed from 100 kPa to 1.6 MPa if initial dia of balloon in 30 cm and relationship of dia of balloon D, when pressure is $p = \frac{C}{D^2}$




***Solution :*** We have

***Free Body Diagram***




Tension = (Bouyant Force – Wt of balloon)

T = (Bouyant Force – 0)

$\Rightarrow$ T = B

$$T = \left(\frac{\pi D^3}{6}\right)\gamma_w \quad ...(i)$$

$$\therefore \quad T \propto D^3 \quad ...(ii)$$

Also $$P \propto \left(\frac{1}{D^2}\right) \quad ...(iii)$$

4

# FLUID KINEMATICS

## THEORY

## 4.1 Definition

Fluid kinematics is defined as the study of fluid motion without considering the effect of force.

**There are two approach are available for fluid analysis**

### *4.1.1 Lagrangian Approach (Microscopic approach)*

This approach is based on the single particle analysis. In this process we are tracking a single fluid particle, so that the result in this process is very accurate.

Such that

$$\vec{u} = \frac{d\vec{x}}{dt},\ \overrightarrow{a_x} = \frac{d\vec{u}}{dt},\ \omega = \frac{d\theta}{dt}$$

**Characteristic of Lagrangian Approach**

(a) P.I.V.: Particle image velocity technique is used in this method.

(b) During fluid flow, it is difficult to keep track of a single fluid particle.

(c) Fluid particle continue deform and does not maintain their identity.

(d) In this method huge time consumption as well as computation time is also high.

### *4.1.2 Eulerian Approach (Macroscopic Approach)*

It is the average approach in this one should not constant on the motion of each and every particle, so we can't give information of different parameters of a particle from this method. Instead of this we get the information of bulk flow.

In this method we should take a control volume in space and find out the variable.

***Example :***

Pressure variable, $P = P(x, y, z, t)$

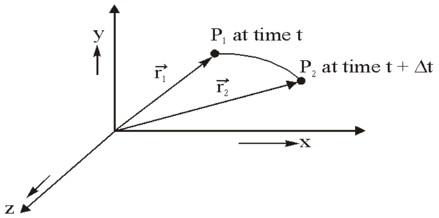
Velocity variable, $\vec{v} = \vec{v}(x, y, z, t)$

Acceleration variable, $\vec{a} = \vec{a}(x, y, z, t)$

In fluid mechanics mostly Eulerian approach is used.

## 4.2 Acceleration Field

There is direct analogy between system and control volume in thermodynamics to Lagrangian versus eulerian description in fluid dynamics. The equation of motion for fluid flow in written for object of fixed identity, taken here as **fluid particle or material particle.** The newton second law to our fluid particle.




$$\vec{F}_{particle} = m_{particle}\,\vec{a}_{particle}$$

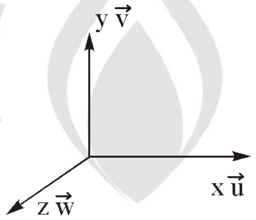
And acceleration of this particle

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt}$$

However velocity of particle is same as local value velocity field at location $\left(x_{particle}(t), y_{particle}(t)\right)$ of the particle since particle moves with fluid.

$$\vec{V}_{particle}(t) = \vec{V}\left(x_{particle}(t), y_{particle}(t), z_{particle}(t), t\right)$$

$$\vec{a}_{particle} = \frac{d\vec{V}_{particle}}{dt} = \frac{d\vec{V}\left(x_{particle}, y_{particle}, z_{particle}, t\right)}{dt}$$




By using Taylor series expansion

$$\vec{a} = \frac{\partial \vec{V}}{\partial t}\frac{dt}{dt} + \frac{\partial \vec{V}}{\partial x_{par}}\frac{dx_{par}}{dt} + \frac{\partial \vec{V}}{\partial y_{par}}\frac{dy_{par}}{dt} + \frac{\partial \vec{V}}{\partial z_{par}} \times \frac{dz_{par}}{dt}$$

From above relation we can find out

$$\vec{a}_x = \frac{\partial \vec{u}}{\partial t} + u\frac{\partial \vec{u}}{\partial x_{par}} + v\frac{\partial \vec{u}}{\partial y_{par}} + w\frac{\partial \vec{u}}{\partial z_{par}}$$

At any instant, the material position vector ($x_{par}$, $y_{par}$, $z_{par}$, t) of fluid particle in Lagrangian frame is equal to position vector (x,y,z) in eulerian frame.

$$\vec{a}_x = \frac{\partial \vec{u}}{\partial t} + u\frac{\partial \vec{u}}{dx} + v\frac{\partial \vec{u}}{\partial y} + w\frac{\partial \vec{u}}{\partial z}$$

Acceleration of fluid particle expressed as field variable

$$\vec{a}(x, y, z, t) = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(\vec{\nabla} \cdot \vec{\nabla}\right)\vec{V} \qquad \text{...(i)}$$

In Cartesian coordinate, component of acceleration vector are

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

Similarly, expression for $a_y$ and $a_z$ can be written.