THEORY & OBJECTIVE

ELECTRICAL MACHINES

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CONTENTS

TRANSFORMERS

THEORY

A Transformer is a static device comprising coupled coils (Primary and Secondary) wound on common magnetic Core.

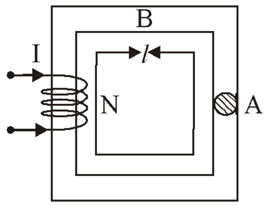
MMF (F)

$$F = NI \text{ Amp. turns}$$

Let the mean core length $= l$

Flux density $= B$, Flux $\phi = BA$

A = Area of cross-section of core.

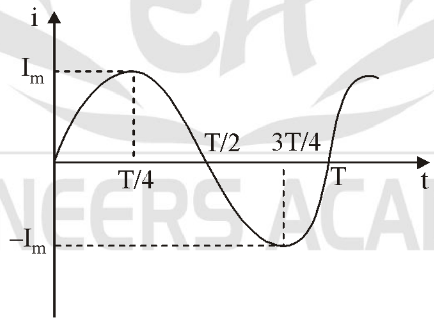


Magnetization force or magnetic field intensity (H)

$$H = \frac{NI}{l} AT^s / m$$

1.1 Magnetic Hysteresis Curve

If ac current is made to flow through coil in the magnetic circuit shown above $i = I_m \sin \omega t$



Time period $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Where $\omega = 2\pi f$ is frequency

Let initially $B = 0$ (i.e. residual magnetism is absent)

For $0 < t < \frac{T}{4}$ where i increases from zero to I_m initially B increases linearly with H (or i) and after a certain value of H, B doesn't increases significantly i.e. B remains almost constant i.e. saturation.

For linear magnetic circuit

$$B \propto H$$

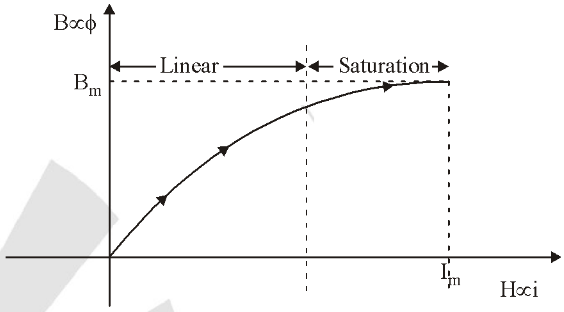
$$\Rightarrow \quad B = \mu H$$

$$\Rightarrow \quad B = \mu_0 \mu_r H$$

Where μ = Permeability of iron core

μ_r = Relative permeability of core

μ_0 = Permeability of air



Flux linking $\phi = BA = \mu HA = \mu A \times \frac{Ni}{l} \quad (H = \frac{Ni}{l})$

$$\Rightarrow \quad \phi = \frac{Ni}{l} = \frac{F}{Rl}$$

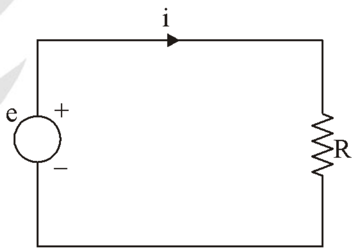
Where F = Ni (mmf in Amp-Turns) applied

$R_l = \frac{l}{\mu A}$ reluctance of core

mmf = Magneto-Motive Force

The equation $\phi = \frac{F}{R_l}$ is developed on basis of the analogy of electrical circuit (force voltage analogy) shown below :

$$\text{Current} = \frac{\text{EMF}}{\text{Resistance}}$$



$$\Rightarrow \quad i = \frac{e}{R}$$

Where resistance $R = \frac{l}{\sigma A} \left(R = \frac{1}{G} = \frac{1}{\sigma A / l} = \frac{l}{\sigma A} \right)$

l & A are the length & area of cross-section while σ is the conductivity of material.

$$\text{Flux} = \frac{\text{mmf}}{\text{Reluctance}}$$

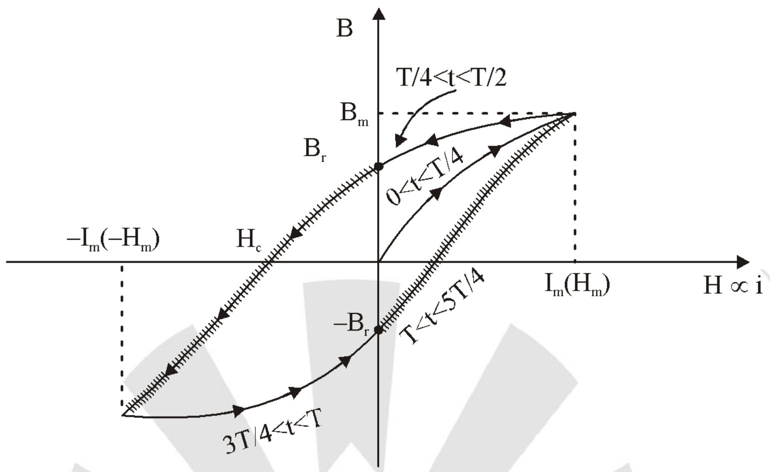
$$\Rightarrow \quad \phi = \frac{F}{R_l}$$

Note : For high permeability material e.g. Iron, μ_r is high & R_l is low it is said to be magnetic conductor or magnetic material. For low permeability material $\mu_r \simeq 1$ e.g. Cu etc. R_l is high so it is said to be non-magnetic material or magnetic insulator.

After $\frac{T}{4}$ i.e. $\frac{T}{4} < t < \frac{T}{2}$ where i decreases from I_m, B also decreases but not in the same manner.

At $t = \frac{T}{2}$, i = 0 but $B = B_r \neq 0$ i.e. some residual magnetism is left.

B_r = Retentivity or Residual flux density.



After $\frac{T}{2}$ i.e. $\frac{T}{2} < t < \frac{3T}{4}$ the direction of current i (& H) gets reversed so magnetization is going on decreasing and at a particular value of current say $I_c \left(\& \ H_c = \frac{NI_c}{l} \right)$ B becomes zero i.e. residual magnetism is lost due to H_c.

H_c = Coercivity or Coercive force

As i increases further (in –ve direction) B gets reversed & becomes max at $t = \frac{3T}{4}$.

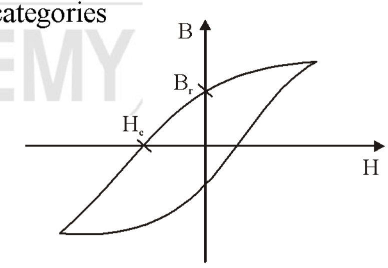
$\Rightarrow$ $$P_h \propto B_m^x f$$

$$P_h = K_h B_m^x f$$

Where x = Steinmetz constant (x = 1.6), K_h = Hystresis coefficient

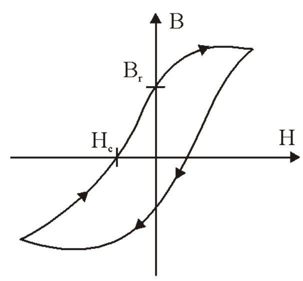
According to B-H loop we categorize magnetic material broadly into two categories

Hard Magnetic Material



- Wider B-H loop
- B_r, H_c (Both high)
- Hysteresis loss Higher
- Suitable for d.c applications & permanent magnet etc.

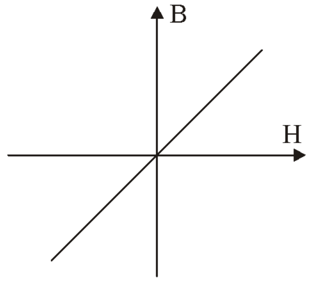
Soft Magnetic Material



- Narrow B-H loop
- B_r, H_c (both low)
- Hysteresis loss small
- Used for a.c applications e.g. Transformer, AC machines.

Let us neglect hysteresis & saturation, the B-H loop will be linear (as in case of air)

According to B-H loop



$$B \propto H$$

$$B = \mu H$$

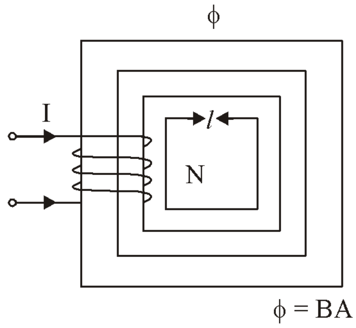
μ = Permeability of core

$$\mu = \mu_0 \mu_r$$

μ_0 = Absolute Permeability

μ_r = Relative Permeability

For linear magnetic circuit $\quad \phi \propto I$



$$\phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{F}{R_l}$$

$$\phi = BA \left(\because B = \frac{\mu NI}{l}\right)$$

$$\phi = \frac{\mu NI}{l} \cdot A$$

$$\phi = \frac{NI}{(l/\mu A)} = \frac{\text{mmf}}{\text{Reluctance}}$$

$$\text{Reluctance} = \frac{l}{\mu A}$$

$$\phi = \frac{\mu NIA}{l}$$

Flux linkage $\quad \psi = N\phi$

$$N\phi \propto I$$

$$N\phi = LI$$

Inductance $\quad L = \frac{N\phi}{I}$ i.e. flux linkage per unit current.

$$L = \frac{N\phi}{I} = \frac{N}{I}\left(\frac{\mu NIA}{l}\right)$$

$$L = \frac{\mu N^2 A}{l}$$

$$\Rightarrow \quad L = \frac{N^2}{(l/\mu A)} = \frac{N^2}{R_l}$$

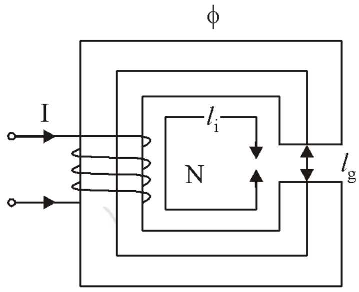
R_l = Reluctance

∵ $$L \propto \frac{1}{R_l}$$

Air gap length $= l_g$

Rl_i = Reluctance of iron path

Rl_g = Reluctance of air path



Total Reluctance in the path of flux, ϕ

$$Rl = Rl_i + Rl_g$$

For iron path $$Rl_i = \frac{l_i}{\mu_0 \mu_r A}$$

$\mu_r \to$ Relative Permeability of iron.

For airgap, $$Rl_g = \frac{l_g}{\mu_0 A}, \text{As } (\mu_r = 1) \text{ for air}$$

As Permeability of iron is much greater than permeability of air ($\mu_r = 1$)

i.e $\mu_r >> 1$

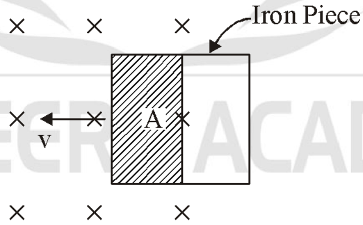
So, there fore we can say Reluctance of air gap will be more.

i.e. $Rl_g >> Rl_i$

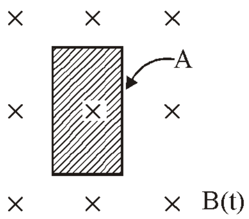
i.e. Total reluctance $Rl \simeq Rl_g$ (air gap reluctance).

1.2 Eddy Currents

If an iron piece is lying in the magnetic field, the flux linking $\phi = BA \cos\theta$, so ϕ can be changed if either B, A or θ changes.



Case-I : B is constant but area 'A' of iron piece linking with B is changing (e.g. in dc machines) i.e. $\phi = BA$ also changing with time.

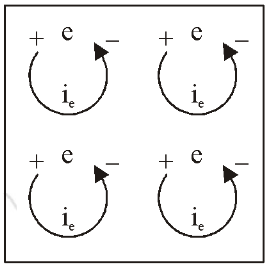


Case-II : Iron piece is stationary but B is changing w.r.t time (e.g. transformers), so ϕ = BA is changing w.r.t time.

As flux linking $\phi(t)$ is changing (in both the cases) there is induced emf in the iron piece i.e.

$$e \propto -\frac{d\phi}{dt}$$

Due to the induced emfs, there are induced currents in the iron i.e. eddy currents i_e



$$i_e = \frac{e}{R_e}$$

Where R_e is the resistance in the path of eddy currents i.e. resistance of iron.

Eddy current loss i.e. power loss due to eddy currents

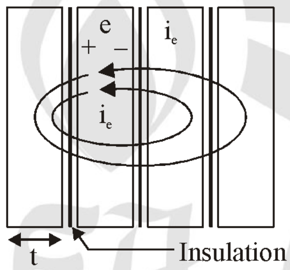
$$P_e = i_e^2 R_e = \frac{e^2}{R_e}$$

As 'e' is independent of R_e

$$\Rightarrow \qquad P_e \propto \frac{1}{R_e}$$

So P_e can be reduced by increasing R_e i.e. using high resistivity iron.

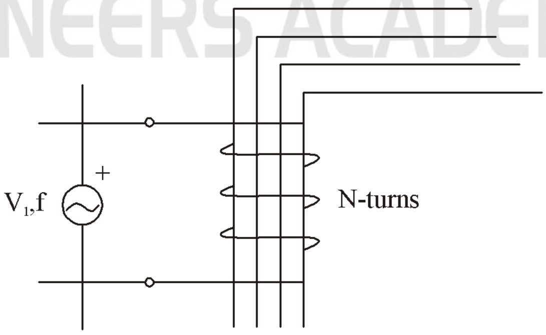
R_e can also be increased if instead of thick iron, laminated iron is used i.e. thin layers of iron pieces are separated by very thin layers of insulation.



As resistance is introduced in the path of eddy currents so resistance R_e increases & hence power loss decreases.

Where 't' is thickness of lamination.

Consider the laminated iron core of transformer



The eddy current losses are

$$P_e \propto \frac{\pi^2 B_m^2 f^2 t^2}{\rho_e \beta}$$

Where B_m is peak flux density in the core

f is frequency

t is thickness of lamination

ρ_e is resistivity of iron core

β is constant (depending upon the shape & size of lamination).

P_e can also be reduced by using high resistance of iron core.

$$P_e \propto B_m^2 f^2 t^2$$

or $$P_e \propto B_m^2 f^2$$

$\Rightarrow$ $$P_e = K_e B_m^2 f^2$$

Where K_e is constant

The combination of hysteresis loss P_h & eddy current loss P_e is said to be iron loss.

$$P_i = P_h + P_e = K_h B_m^{1.6} f + K_e B_m^2 f^2$$

1.3 Transformer Equation

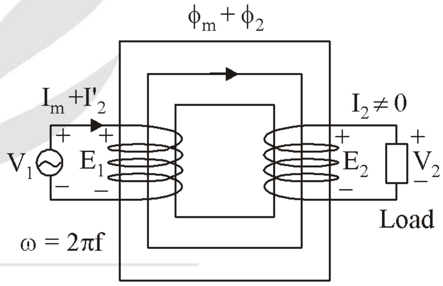
Primary : Where source is connected.

Secondary : Where load is connected.

At No load. Due to magnetising current I_m magnetising flux ϕ_m is produced.

Let $\phi_m = \phi_{max} \sin\omega t$

Induced emf in Primary and secondary is e_1 & e_2



$$e_1 = \frac{-N_1 d\phi_m}{dt}, \; e_2 = \frac{-N_2 d\phi_m}{dt}$$

$$e_1 = -N_1 \frac{d}{dt}(\phi_{max} \sin\omega t)$$

$$e_1 = -N_1 \omega \phi_{max} \cos\omega t$$

$$e_1 = -N_1 \omega \phi_{max} \sin(90 - \omega t),$$

As $$e_1 = N_1 \omega \phi_{max} \sin(\omega t - 90°)$$

peak emf $$E_m = N_1 \omega \phi_{max}$$

We can say induced emf $\bar{E}$ lags behind the corresponding flux ϕ_m by 90°

$$e_i = N_1 \omega \phi_{max} \sin(\omega t - 90°)$$

$$e_i = E_m \sin(\omega t - 90°)$$

Peak emf $$E_m = N_1 \omega \phi_{max}$$

$$= N_1 (2\pi f) \phi_{max} = 2\pi f N_1 \phi_{max}$$

Let r.m.s value of E_m is E_1 $$E_1 = \frac{E_m}{\sqrt{2}} = \sqrt{2}\pi f N_1 \phi_{max}$$

Similarly $E_2 = \sqrt{2}\pi N_2 f\,\phi_{max}$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$\frac{N_2}{N_1} = K$$

Turns ratio or Transformation ratio

As $V_1 \approx E_1$

$$V_1 \simeq \sqrt{2}\pi f\, N_1\phi_{max}$$

$$\phi_{max} = \frac{1}{\sqrt{2}\pi N_1}\frac{V_1}{f}$$

$$\phi_{max} = \frac{1}{\sqrt{2}\pi N_1}\left(\frac{V_1}{f}\right)$$

$$\phi_{max} \propto \frac{V_1}{f}$$

$$\phi_{max} = \text{constant}$$

$$\phi_m = \frac{N_1 I_m}{Rl} \text{ \& Secondary flux } \phi_2 = \frac{N_2 I_2}{Rl}$$

According to Lenz's law the flux ϕ_2 of current I_2 will oppose ϕ_m.

Lenz's law : The induced current flows in such a direction so as to oppose very cause of its production.

So net flux in core = $\phi_m - \phi_2$

Due to flux ϕ_2 net flux in the core decreased, $(\phi_m - \phi_2)$

However $\phi_m \propto \frac{V_1}{f} = \text{constant},$

That's why to maintain flux ϕ_m constant the primary winding produces additional flux ϕ_2, so it takes additional current I'_2

Secondary flux $\phi_2 = \frac{N_2 I_2}{R_l}$

Additional flux by Primary $\phi_2 = \frac{N_1 I'_2}{R_l}$

$\therefore$ $\phi_2 = \frac{N_1 I'_2}{R_l} = \frac{N_1 I_2}{R_l}$

$$N_1 I'_2 = N_2 I_2$$

Primary Current $\bar{I}_1 = \bar{I}_m + \bar{I}'_2$

As $\bar{I}_m << \bar{I}'_2$

$I_1 \simeq I'_2$

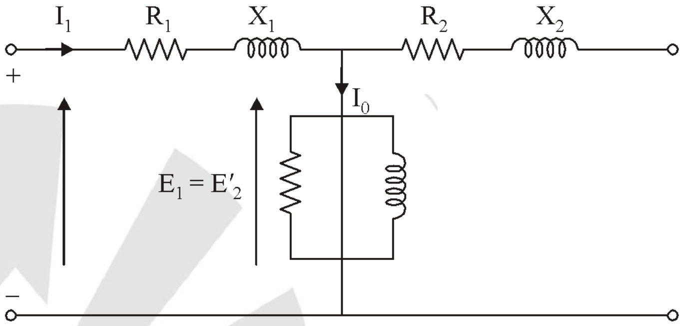
so $N_1 I_1 \simeq N_2 I_2$

This equation is valid only when magnetising current is negligible.

Example 1

Note : $E_1 \simeq V_1$

$V_1 = \bar{E}_m + \bar{I}_m(R_1 + jX_1)$

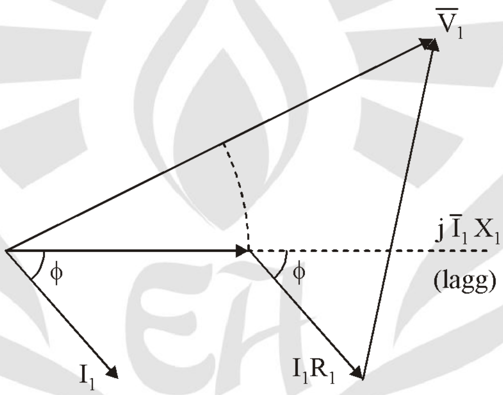


Solution

Let at lag pf cos ϕ

$\bar{I}_1$ lags $\bar{E}_1$ by angle ϕ

At lagging

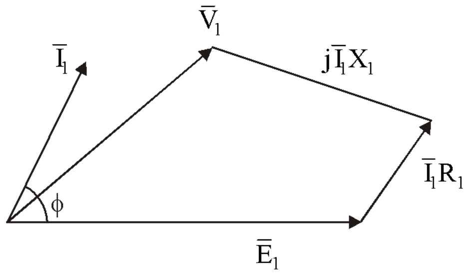


So $E_1 \le V_1$ (As leakage impedance R_1, X_1 very low)

At leading pf cos ϕ

$\bar{I}_1$ leads $\bar{E}_1$ by angle ϕ

$E_1 \ge V_1$



Example 2 : The useful flux of a Transformer is 1Wb. when it is loaded at 0.8pf lag, then its mutual flux

(a) May decrease to 0.8Wb
(b) May increase to 1.01Wb
(c) Remains constant
(d) May decrease to 0.99Wb

Ans.(d)

Solution: At no load (I = 0)

$$V_1 \approx E_1$$

$$E_1 = \sqrt{2}\pi f N_1 \phi_m$$

$$\phi_m = \frac{1}{\sqrt{2}\pi N_1}\frac{E_1}{f}$$

At no load $E_1 \approx V_1$

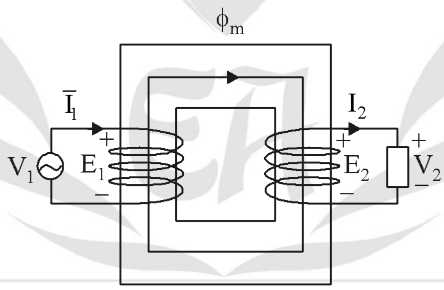
$$\phi_{mo} = \frac{1}{\sqrt{2}N_1\pi}\left(\frac{V_1}{f}\right) = 1\text{Wb}$$

At lag pf E_1 decreases slightly ie $E_1 \leq V_1$

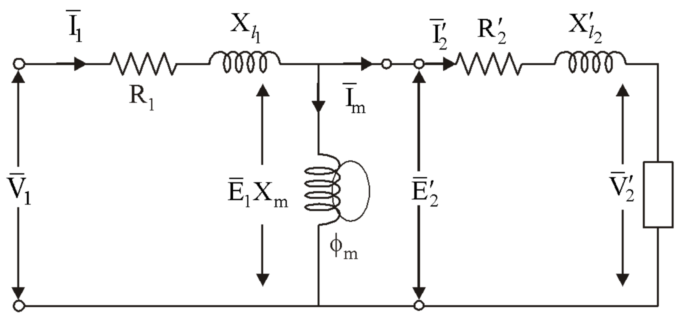
So $$\phi'_m = \frac{1}{\sqrt{2}\pi N_1}\left(\frac{E_1}{f}\right) \leq \phi_{mo}$$

So magnetising flux decreases slightly ie approx 0.99 Wb

Equivalent Circuit of Transformer



ϕ_m = magnetising flux or useful flux



R_1 = Resistance of Primary

R_2 = Resistance of Secondary

X_{l_1} = Leakage Reactance of Primary

X_{l_2} = Leakage Reactance of Secondary

X_m = Magnetising Reactance

Leakage flux through air = ϕ_l

$\because \quad \phi_m >> \phi_l$

Total flux $\quad \phi = \phi_m + \phi_l$

$L = L_m + L_l$ (because $N\phi = LI$)

$\phi \propto L$

Where $\quad L_m$ = Magnetising inductance

L_l = Leakage inductance

$L = L_m + L_l$

Multiplying both side by ω

$$\omega L = \omega L_m + \omega L_l$$

$$X = X_m + X_l$$

X_m = Leakage Reactance

X_l = Magnetising Reactance

Magnitude of e

$$e = \underbrace{N\frac{d\phi}{dt}}_{\text{induced emf}} = \underbrace{L\frac{di}{dt}}_{\text{voltage drop across L}}$$

From the equivalent circuit we can write

$$E_2 = V_2 + I_2(R_2 + jX_{12})$$

$$\frac{E_2}{K} = \frac{V_2}{K} + (KI_2)\left(\frac{R_2}{K^2} + j\frac{X_{12}}{K^2}\right)$$

$$E_2' = V_2' + I_2'(R_2' + jX_{l_2}')$$

Compare above equations :

$E_2' = \frac{E_2}{K}$ = Secondary emf referred to Primary

$V_2' = \frac{V_2}{K}$ = Secondary voltage referred to Primary

$I_2' = KI_2$ = Secondary current referred to Primary

$R_2' = \frac{R_2}{K^2}$ = Secondary resistance referred to Primary

$$X'_{l_2} = \frac{X_{l_2}}{K^2} = \text{Secondary leakage Reactance referred to Primary.}$$

The iron losses or core losses of iron core

$$P_i = P_h + P_e$$

Where $P_h \rightarrow$ Hysteresis loss

$P_e \rightarrow$ Eddy current loss

$$P_h = K_h B_m^{1.6} f \ \ \& \ \ P_e = K_e B_m^2 f^2$$

$B_m \rightarrow$ Peak flux density,

$f \rightarrow$ Frequency

Magnetising flux

$$\phi_m = \frac{1}{\sqrt{2}\pi N_1}\left(\frac{E_1}{f}\right)$$

$$\Rightarrow \quad B_m = \frac{1}{\sqrt{2}\pi N_1 A}\left(\frac{E_1}{f}\right)$$

Let $P_e \propto E_1^2$ (approximated)

& $P_e \propto E_1^2$

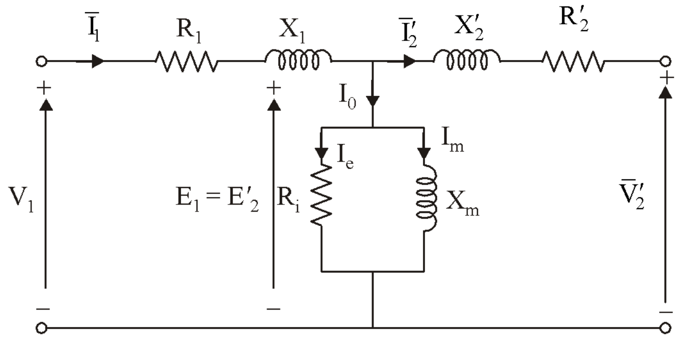
So $P_i \propto E_1^2$

Hence $P_i = \dfrac{E_1^2}{R_i}$

where $R_i \rightarrow$ is constant

$R_i \rightarrow$ core loss resistor

By connecting R_i across voltage E_1 in equivalent circuit, iron losses can be represented



$$\bar{I}_1 = \bar{I}_0 + \bar{I}'_2$$

Where no load current $\bar{I}_0 = \bar{I}_e + \bar{I}_m$

$\bar{I}_0$ is approximately 4-5% of rated or full load current.

$$\bar{E}_1 = \bar{V}_1 - \bar{I}_1(R_1 + jX_1)$$

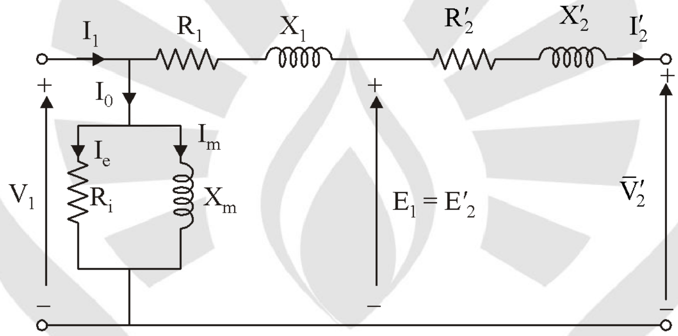
Let primary leakage impedance drop is neglected

$$E_1 \simeq V_1 \text{ so no load current}$$

$$\bar{I}_0 = \frac{\bar{E}_1}{\bar{Z}_0} \simeq \frac{\bar{V}_1}{\bar{Z}_0}$$

Where $\bar{Z}_0$ is the impedance of magnetising branch

Hence $\bar{Z}_0$ can also be connected across $\bar{V}_1$ so approximated equivalent circuit is

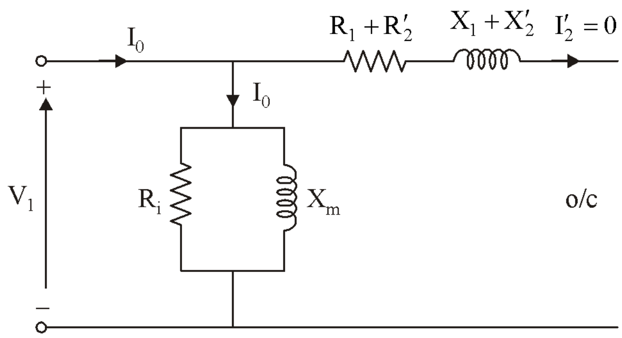


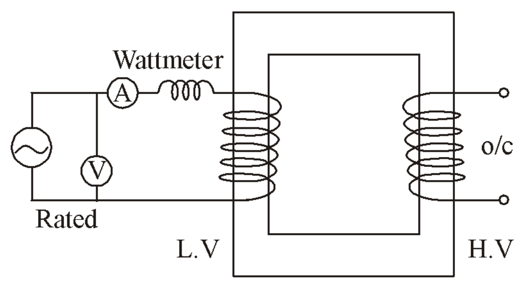
1.4 Open Circuit and Short Circuit Test

These test are performed to determine the circuit constants, efficiency and regulation without actually loading the Transformer.

Open Circuit Test or No Load Test

To determine iron loss, As iron loss depends upon the applied voltage so it is performed at rated voltage hence it is performed on L.V side





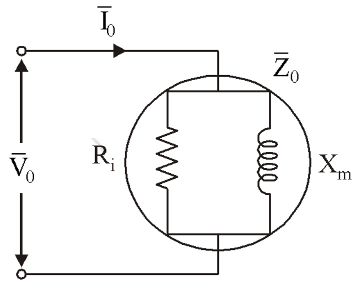
Readings

$V_0 \quad I_0 \quad P_i$

Where V_0 is the rated voltage applied.

$$P_i = \frac{V_0^2}{R_i}$$

$$\Rightarrow \qquad R_i = \frac{V_0^2}{P_i}$$



$$G_i = \frac{1}{R_i}$$

$$\frac{1}{Z_0} = \frac{1}{R_i} + \frac{1}{jX_m} = \frac{1}{R_i} - j\frac{1}{X_m}$$

$$\bar{Y}_0 = G_i - jB_m$$

$\bar{Y}_0$ admittance

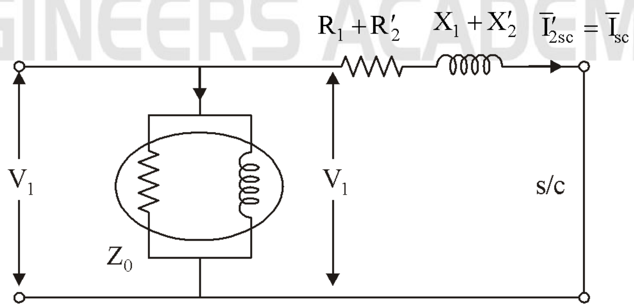
$$|Y_0| = \sqrt{G_i^2 + B_m^2}$$

$$Z_0 = \frac{V_0}{I_0} \text{ and } Y = \frac{I_0}{V_0}$$

Susceptance $\quad B_m = \sqrt{Y_0^2 - G_i^2}$

Reactance $\quad X_m = \frac{1}{B_m}$

1.5 Short Circuit Test or S.C. Test



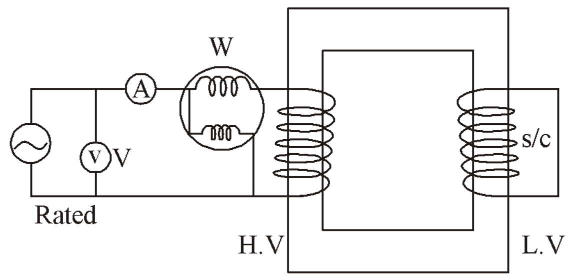
Short circuit test is performed to determine Cu-loss, As Cu-loss depends upon load current so short circuit test is performed at rated current.

To flow the rated current in short circuit condition reduced voltage upto 5% of rated voltage is required.

So short circuit test is performed at H.V side or low current side.

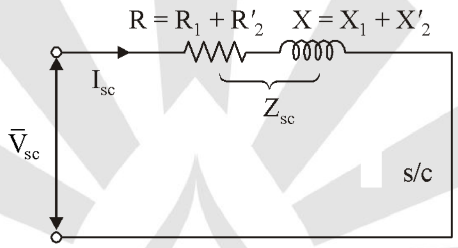
Reduced Voltage is Applied for SC Test

At reduced voltage the no load current I_0 will be very very small, so it can be neglected.



$$Z_{sc} = R + jX$$

$$Z_{sc} = \sqrt{R^2 + X^2}$$



Reading

$$V_{sc} \quad \underbrace{I_{sc}}_{\text{rated value}} \quad \underbrace{P_{sc}}_{\text{Cu-loss}}$$

$$P_{sc} = I_{sc}^2 R \quad \Rightarrow \quad R = \frac{P_{sc}}{I_{sc}^2}$$

$$R = \frac{P_{sc}}{I_{sc}^2}$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

$$X = \sqrt{Z_{sc}^2 - R^2}$$

1.6 Transformer Efficiency

The ratio of output power to the input power in a machine is known as the efficiency.

$$\eta = \frac{P_0}{P_{in}} = \frac{P_0}{P_0 + P_L}$$

Power input = Power output + Losses

$$P_{in} = P_0 + \text{Losses}$$

$$\% \, \eta = \frac{\text{Output}}{\text{Output} + \text{Losses}} \times 100$$