THEORY & OBJECTIVE

HEAT & MASS TRANSFER

By
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CONTENTS

1 INTRODUCTION

THEORY

1.1 INTRODUCTION

Heat is the form of energy that can be transferred from one system to another as result of temperature difference. Heat transfer deals with system that take thermal equilibrium and thus it is non-equilibrium phenomenon.

The energy can exist in numerous forms such as thermal, mechanical, electric, magnetic, chemical and nuclear and their sum constitute the total energy E of the system. The form of energy related to molecular structure of system and degree of molecular activity is referred to as internal energy. Internal energy can be viewed as sum of kinetic and potential energies of the molecules. The portion of internal energy associated with kinetic energy of molecules is called sensible energy or sensible heat. The internal energy associated with inter molecular forces between molecules of as system is called latent energy or latent heat.

1.2 ENERGY TRANSFER

Energy can be transferred to or from a given mass by two mechanism: heat transfer Q and work W. The sensible and latent form of internal energy are termed as thermal energy. The transfer of thermal energy is heat transfer.

Since, first law of thermodynamic says,

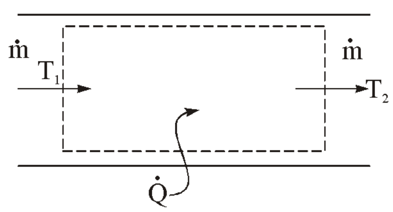
$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work or mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Rate of change in} \\ \text{Internal, kinetic, potential etc.}}}$$

In heat transfer analysis, we consider only that form of energy which can be transferred as a result of temperature difference i.e., thermal energy. The conversion of nuclear, chemical, mechanical and electrical energies into thermal energy is denoted by heat generation. i.e.,

$$\dot{Q}_{in} - \dot{Q}_{out} + \dot{Q}_{gen} = \frac{dE}{dt}$$

Thermal, system for steady flow system,

$$\dot{Q} = \dot{m}\Delta h = \dot{m}C_P\Delta T$$

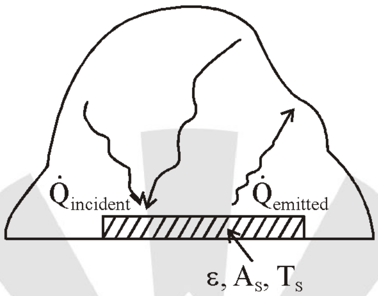


$$\dot{Q}_{absorbed} = \alpha \dot{Q}_{incident}$$

But, according to Kirchoff law the emissivity and absorptivity of surface at given temperature are equal.

$$\dot{Q}_{abs} = \varepsilon \dot{Q}_{incident}$$

When surface of emissivity ε and surface area A_s is completely enclosed by much larger (or black) surface at thermodynamic temperature $T_{surrounding}$ separated by gas that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is



$$\dot{Q}_{rad} = \varepsilon A_s \sigma \left(T_s^4 - T_{surr}^4\right) (\because \alpha = \varepsilon)$$

There are three mechanisms of **heat transfer,** but not all three can exist simultaneously in a medium. **Heat transfer** is only by conduction in opaque solids, but by conduction and radiation in semitransparent solids. Thus, solid may involved **conduction** and **radiation** but not **convection.** How ever, a solid may involve heat transfer by convection and/or radiation at its surface exposed to fluid or other surfaces. For example, outer surface of solid piece of rock will warm up in warmer environment as result of heat gain by convection (from the air) and the radiation (from the sun and warmer surrounding surfaces). But inner part of rock will warm up as heat is transferred to inner region of rock by conduction.

Heat transfer is by conduction and by radiation in still fluid (no bulk fluid motion) and by convection and radiation in a flowing fluid.

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2

HEAT CONDUCTION

THEORY

General relation for Fourier Law of heat conduction,

$$\dot{Q}_n = -KA\frac{\partial T}{\partial n}$$

Where n = normal of isothermal surface at point P.

In rectangular co-ordinates, the heat conduction vector can be expressed in terms of its components as

$$\vec{\dot{Q}}_n = \vec{\dot{Q}}_x\hat{i} + \dot{Q}_y\hat{j} + \dot{Q}_z\hat{k}$$

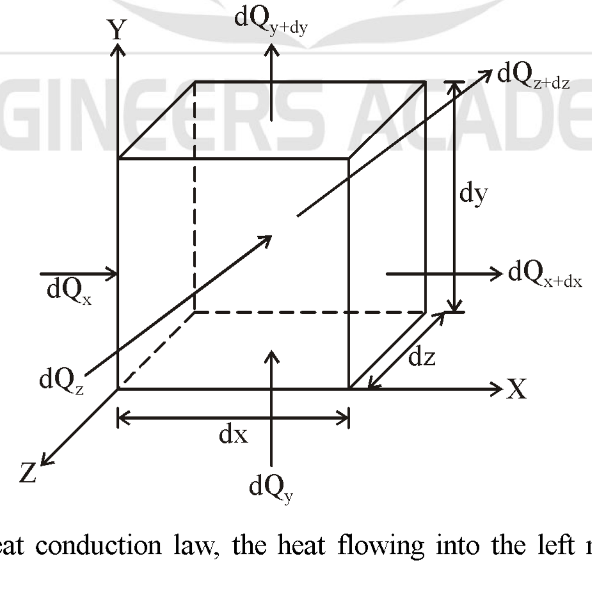
Where, $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors and $\dot{Q}_x, \dot{Q}_y$ and $\dot{Q}_z$ are the magnitude of heat transfer rates in x, y and z-directions which can be determined by Fourier Law as

$$\dot{Q}_x = -KA_x\frac{\partial T}{\partial x},\quad \dot{Q}_y = -KA_y\frac{\partial T}{\partial y},\quad \dot{Q}_z = -KA_z\frac{\partial T}{\partial z}$$

2.1 General Heat Conduction Equation

Let us consider an infinitesimal volume element of side dx, dy and dz.

Now consideration here will include the non steady condition of temperature variation with time t.



According to Fourier heat conduction law, the heat flowing into the left most face of the element in the X-direction.

$$dQ_x = -K.dy.dz.\frac{\partial T}{\partial x}$$

From Taylor's series

$$dQ_{x+dx} = dQ_x + \frac{\partial}{\partial x}(dQ_x).dx$$

The net heat flow by conduction in X-direction.

$$dQ_x - dQ_{x+dx} = -\frac{\partial}{\partial x}(dQ_x).dx$$

$$= -\frac{\partial}{\partial x}\left(-K.dy.dz.\frac{\partial T}{\partial x}\right).dx$$

$$= K.dx.dy.dz.\frac{\partial^2 T}{\partial x^2} \quad ...(1)$$

Similarly in Y-direction and z-direction

$$dQ_y - dQ_{y+dy} = K.dx.dy.dz.\frac{\partial^2 T}{\partial y^2} \quad ...(2)$$

$$dQ_z - dQ_{z+dz} = K.dx.dy.dz.\frac{\partial^2 T}{\partial z^2} \quad ...(3)$$

Let q_g is the rate at which heat is generated initially per unit volume.
Then the total rate of heat generation in elemental volume is = $q_g.dxdy.dz$ ...(4)

The rate of accumulation of internal energy within the control volume = $mC\frac{\partial T}{\partial t}$

$$= \rho.dxdy.dz\frac{\partial T}{\partial t} \quad ...(5)$$

From energy balance equation

Rate of energy storage within the solid = Rate of heat influx – Rate of heat outflux + Rate of heat generation

$$\Rightarrow \quad \rho c.dxdy.dz.\frac{\partial T}{\partial t} = (dQ_x + dQ_y + dQ_z) - (dQ_{x+dx} + dQ_{y+dy} + dQ_{z+dz}) + q_G.dxdy.dz$$

$$\Rightarrow \quad \rho.C.\frac{\partial T}{\partial t} = K\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right\} + q_G$$

$$\Rightarrow \quad \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{K} = \frac{\rho C}{K}.\frac{\partial T}{\partial t}$$

$$\Rightarrow \quad \nabla^2 T + \frac{q_G}{K} = \frac{1}{\alpha}\cdot\frac{\partial T}{\partial t}$$

Where $\alpha \rightarrow$ thermal diffusivity

In cylindrical co-ordinate

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r.\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}.\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_G}{K} = \frac{1}{\alpha}.\frac{\partial T}{\partial t}$$

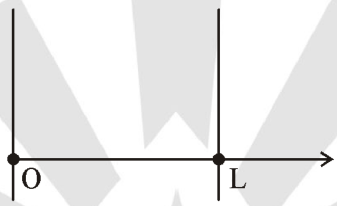
In spherical co-ordinate

$$\frac{1}{r^2}.\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}.\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}+\frac{q_G}{K} = \frac{1}{\alpha}.\frac{\partial T}{\partial t}$$

2.1.1 Boundary and Initial Condition

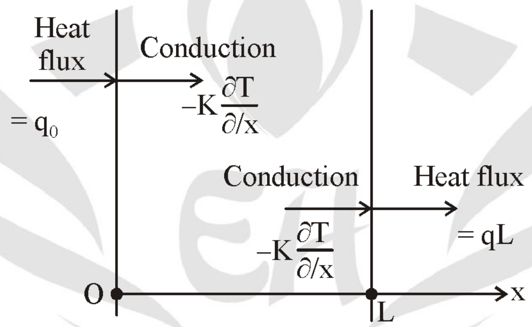
To describe a heat transfer problem completely, two boundary condition must be given for each direction of coordinate system along which heat transfer is significant.

(i) Specified temperature boundary condition : (Drichilet Boundary Condition)



$T(0, t) = T_1$ and $T(L, t) = T_2$

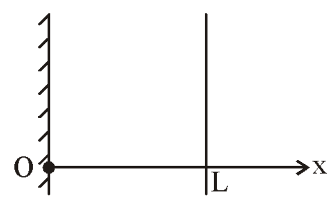
(ii) Specified heat flux boundary condition : For a plate of thickness L subjected to heat flux of 50 W/m^2 into medium from both side, the specified flux boundary conditions are



$$-K\frac{\partial T}{\partial x}(L,t) = -50$$

Since flux at surface x = L is in negative x-direction, thus it is – 50 W/m^2.

Special case (1) : Insulated boundary

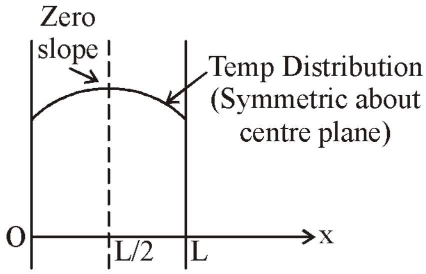


$$\dot{q} = 0 \Rightarrow -K\frac{\partial T}{\partial x}(0,t)=0$$

$$\Rightarrow \quad \frac{\partial T}{\partial x}(0,t) = 0$$

That is, on an insulated surface, the first derivative of temperature w.r.t. space variable (the temperature gradient) in direction normal to insulated surface is zero. This means temperature function must be perpendicular to insulated surface since slope of temperature at surface must be zero.

Special case (2) : Thermal Symmetry



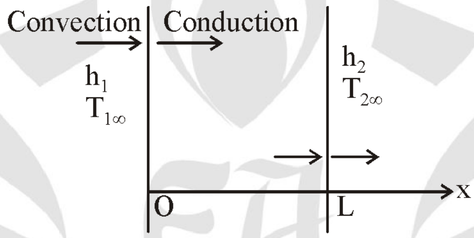
It possesses thermal symmetry about centre plane at

$$x = \frac{L}{2}$$

$$\frac{\partial T}{\partial x}\left(\frac{L}{2}, t\right) = 0$$

(iii) Convectional boundary condition : The convection boundary condition is based on surface energy balance

Heat conduction at surface in a selected direction = Heat convection at surface in the same direction

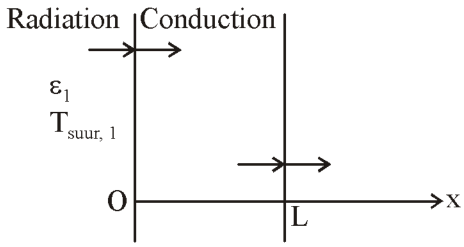


$$-K\frac{\partial T(0,t)}{\partial t} = -\,h_1\left[T_{\infty 1} - T_{(0,t)}\right]$$

and $$-K\frac{\partial T(L,t)}{\partial t} = h_1\left[T(L,t) - T_{\infty 2}\right]$$

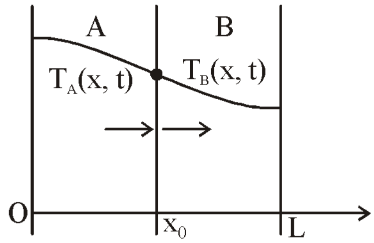
(iv) Radiation boundary condition :

Heat conduction at surface in a selected direction = Radiation exchange at surface in the same direction



$$-K\frac{\partial T(0,t)}{\partial x} = \varepsilon_1\sigma\left[T_{surr,1}^4 - T_{(0,t)}^4\right]$$

(v) Interface boundary condition : The boundary condition at interface are based on requirement



(a) Two bodies in contact must have same temperature at area of contact.

(b) An interface cannot store energy, thus heat flux on both side of interface must be same.

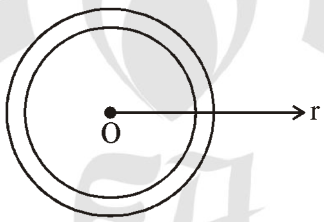
$$T_A\ (x_A,\ t) = T_B\ (x_0,\ t)$$

$$-K_A \frac{\partial T_A(x_0, t)}{\partial x} = -K_B \frac{\partial T_B(x_0, t)}{\partial n}$$

Example 1 : Consider a spherical container of inner radius r_1 = 8 cm and r_2 = 10 cm, K = 45 W/m°C. The inner and outer surface of container are maintained at temperature of T_1 = 200°C and T_2 = 80°C. Determine general relation for temperature distribution inside the shell under steady condition and determine the rate of heat loss from the container.

Solution :

Heat transfer is steady the heat transfer is one-dimensional since there is thermal symmetry about mid point.



$$\left.\frac{dT}{dr}\right|_{r=0} = 0$$

So isotherm are concentric sphere. So T = T(r)

Heat conduction equation,

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

$$\frac{dT}{dr} = \frac{c_1}{r^2}$$

$$T(r) = \frac{c_1}{r} + c_2$$

Boundary condition are,

$$T(r) = T_1 = 200°C$$

$$T(r_2) = T_2 = 80°C$$

$$T_1 = \frac{-c_1}{r_1} + c_2 \text{ and } T_2 = \frac{-c_1}{r_2} + c_2$$

Solving for c_1 and c_2 and substituting for T,

$$T(r) = \frac{r_1 r_2}{r(r_2 - r_1)}(T_1 - T_2) + \frac{r_2 T_2 - r_1 T_1}{r_2 - r_1}$$

The rate of heat loss from container is rate of heat conduction through container wall,

$$\dot{Q}_{shpere} = -kA\frac{dT}{dr} = +k(4\pi r^2).\frac{r_1 r_2 (T_1 - T_2)}{r^2 (r_2 - r_1)} = \frac{4\pi k r_1 r_2 (T_1 - T_2)}{r_2 - r_1}$$

2.2 Heat Generation in a Solid

Heat generation is expressed per unit volume of medium

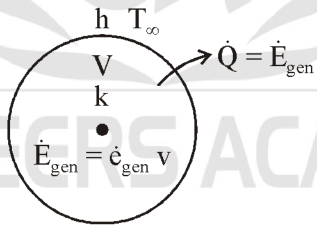
$$\dot{e}_{gen} = \frac{\dot{E}_{gen,electric}}{V_{wire}} = \frac{I^2 Re}{\pi r_0^2 L}$$

The temperature of medium rises during heat generation at start up condition. At steady state, the rate of heat generation equals the rate of heat transfer to surroundings.

The maximum temperature T_{max} in a solid that involves uniform heat generation occur at a location farthest away from outer surface when outer surface of solid is maintained at constant temperature T_s. For example, maximum temperature occur at mid plane in sphere. The temperature distribution within solid in these cases is symmetrical about centre of symmetry.

Consider a solid medium of surface area A_s and volume V and volume V and constant thermal conductivity, where heat is generated at $\dot{e}_{gen}$ per unit volume under steady condition.

Rate of heat transfer from the solid= Rate of heat generation within the solid



$$\dot{Q} = \dot{e}_{gen} v$$

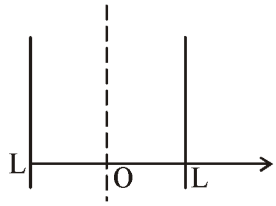
$$\Rightarrow \quad hA_s (T_s - T_\infty) = \dot{e}_{gen} v$$

$$\Rightarrow \quad T_s = T_\infty + \frac{\dot{e}_{gen} v}{hA_s}$$

For large plane wall, of thickness 2L

$$A_s = 2 A_{wall}$$

and $$v = 2 A_{wall} L$$

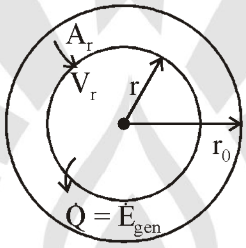


$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{e}_{gen} L}{h}$$

For cylinder, $A_s = 2\pi r_0 L,\ v = \pi R_0^2 L$

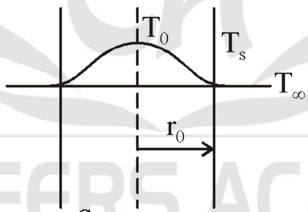
$$T_{s,\text{ cyl}} = T_\infty + \frac{\dot{e}_{gen} r_0}{3h}$$

Consider heat transfer from long solid cylinder. The heat generated within this inner cylinder must be equal to heat conducted through outer surface.



$$-kA_r \frac{dT}{dr} = \dot{e}_{gen} V_r$$

Where $dT = -\dfrac{\dot{e}_{gen}}{2k} r\, dr$



Integrating At $r = 0,\ T = T_0$

$r = r_0,\ T = T_s$

$$T_s - T_0 = -\frac{\dot{e}_{gen}}{2k}\frac{r_0^2}{2} = -\frac{\dot{e}_{gen} r_0^2}{4k}$$

$$\Rightarrow \quad T_s - T_0 = \frac{\dot{e}_{gen} r_0^2}{4k}$$

$$\Rightarrow \quad \Delta T_{max} = T_0 - T_s = \frac{\dot{e}_{gen} r_0^2}{4k}$$

For plane wall, $\Delta T_{max} = \dfrac{\dot{e}_{gen} L^2}{2k}$

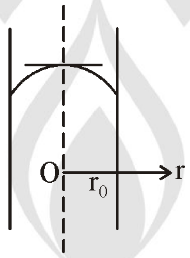
For sphere, $\Delta T_{max} = \dfrac{\dot{e}_{gen} r_0^2}{6k}$

Example 2 : A long homogenous resistance wire of radius r_0 = 0.5 cm and thermal conductivity k = 13.5 W/m°C is being used to boil water at atmosphere pressure by passage of electric current. Heat is generated in wire uniform *l*y as result of resistance heating at the rate of $\dot{e}_{gen}$ = 4.3 × 10^7 W/m^3. If the outer surface temperature of wire is measured to be T_s = 180°C, obtain relation for temperature distribution, and determine temperature at centre line of wire when steady operating condition are reached.

Solution :

Heat conduction equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{e}_{gen}}{k} = 0$$



Two boundary condition are

$$\frac{dT(0)}{dr} = 0,\ T(r_0) = T_s = 108°C$$

Integrating above equation

$$r\frac{dT}{dr} = -\frac{\dot{e}_{gen} r^2}{2k} + c_1$$

$$T = -\frac{\dot{e}_{gen} r^2}{4k} + c_1 \ln r + c_2$$

Applying first condition $c_1 = 0$

$$T = T_s + \frac{\dot{e}_{gen}}{4k}\left(r_0^2 - r^2\right)$$

Maximum temperature occur at centre line r = 0

$$T = T_s + \frac{\dot{e}_{gen}}{4k} r_0^2$$

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OBJECTIVE QUESTIONS

1. Thermal conductivity is lower for

(a) wood (b) air

(c) water at 100°C (d) steam at 1 bar

2. Match the property with their units

PROPERTY

A. Bulk modulus

B. Thermal conductivity

C. Heat transfer coefficient

D. Heat flow rate

UNITS

1. W/s
2. N/m^2
3. N/m^3
4. W
5. W/mK
6. W/m^2K

3. Consider the following statements :

1. Temperature of the surface.
2. Emissivity of the surface.
3. Temperature of the air in the room.
4. Length and diameter of the pipe.

The parameter(s) responsible for loss of heat from at hot surface in a room would include

(a) 1 only (b) 1 and 2

(c) 1, 2 and 3 (d) 1, 2, 3 and 4

4. For a given heat flow and for the same thickness, the temperature drop across the material will be maximum for

(a) Copper (b) Steel

(c) Glass wool (d) Refratory brick

5. Heat is mainly transferred by conduction, convection and radiation in

(a) insulated pipes carrying hot water

(b) refrigerator freezer coil

(c) boiler furnaces

(d) condensation of steam in a condenser

6. Match List (Law) with List-II (equation) and select the correct answer using the codes given below the lists:

List-I

A. Stefan-Boltzmann law

B. Newton's law of cooling

C. Fourier's law

D. Kirchoff's law

List-II

1. $q = hA(T_1 - T_2)$
2. $E = \alpha E_b$
3. $q = \dfrac{kA}{L}(T_1 - T_2)$
4. $q = \sigma A(T_1^4 - T_2^4)$
5. $q = kA(T_1 - T_2)$

Codes :

	A	B	C	D
(a)	4	1	3	2
(b)	4	5	1	2
(c)	2	1	3	4
(d)	2	5	1	4

7. In descending order of magnitude, the thermal conductivity of (a) Pure iron, (b) liquid water, (c) Saturated water vapour, (d) Pure aluminum can be arranged as

(a) a, b, c, d (b) b, c, a, d

(c) d, a, b, c (d) d, c, b, a

8. In case of one dimensional heat conduction in a medium with constant properties, T is the temperature at position x, at time t. Then $\dfrac{\partial T}{\partial t}$ is proportional to

(a) $\dfrac{T}{x}$ (d) $\dfrac{\partial T}{\partial x}$

(c) $\dfrac{\partial^2 T}{\partial_x \partial t}$ (d) $\dfrac{\partial^2 T}{\partial x^2}$

9. A 100W electric bulb was switched on in a 2.5m × 3m × 3m size thermally insulated room having temperature of 20ºC. Room temperature at the end of 24 hours will be

(a) 321°C (b) 341°C

(c) 450°C (d) 470°C

Statement for linked answer question 12 & 13

Consider steady one-dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m^3. The left and right faces are kept at constant temperatures of 160°C and 120°C respectively. The plate has a constant thermal conductivity of 200 W/m.K.

10. The location of maximum temperature within the plate from left face is

(a) 15 mm (b) 10 mm

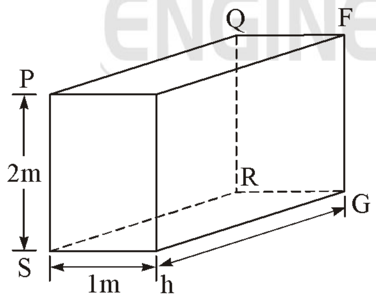
(c) 5 mm (d) 0 mm

11. The maximum temperature within the plate in degree C is

(a) 160 (b) 165

(c) 175 (d) 250

12. For the three dimensional object shown in the fig below. Five faces are insulated. The sixth face (PQRS), which is not insulated, interacts thermally with the ambient, with a convective heat transfer coefficient of 10 W/m^2K. the ambient temperature is 30°C. heat is uniformly generated inside the object at the rate of 100 W/m^3. assuming the face PQRS to be at uniform temperature, its steady state temperature is



(a) 10°C (b) 20°C

(c) 30°C (d) 40°C

13. In MLTθ system (T being time and θ temperature), what is the dimension of thermal conductivity?

(a) $ML^{-1}T^{-1}\theta^{-3}$ (b) $ML^{-1}\theta^{-1}$

(c) $ML\theta^{-1}T^{-3}$ (d) $ML\theta^{-1}T^{-2}$

14. In which one of the following materials, is the heat energy propagation minimum due to conduction heat transfer ?

(a) Lead (b) Copper

(c) Water (d) Air

15. A plane wall of thickness 2L has a uniform volumetric heat source q* (W/m^3). It is exposed to local ambient temperature T_∞ at both the ends (x = ± L). The surface temperature t_s of the wall under steady-state condition (where h and k have their usual meanings) is given by

(a) $T_s = T_\infty + \frac{q^* L}{h}$ (b) $T_s = T_\infty + \frac{q^* L}{2k}$

(c) $T_s = T_\infty + \frac{q^* L^2}{h}$ (d) $T_s = T_\infty + \frac{q^* L^3}{2k}$

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