

THEORY & OBJECTIVE

# POWER SYSTEMS

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## MODELLING OF TRANSMISSION LINE

### 1.1 PARAMETERS PERFORMANCE

A particular conductor of cross-sectional area 'A' and length 'l' having resistance R.

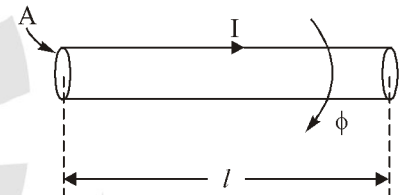
$$R = \rho \frac{l}{A}$$

⇒

$$R = \frac{l}{\sigma A}$$

$\rho$  → Resistivity

$\sigma$  → Conductivity



Whenever a current is passed through a conductor it produces a flux 'ϕ'.

Where,

$$\phi \propto I$$

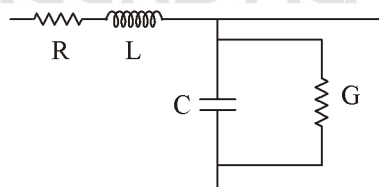
⇒

$$\phi = LI$$

So there exists an inductance also

$$L = \frac{\phi}{I}; L \rightarrow \text{Inductance}$$

There is some capacitance exists between two conductors where air behaves as insulator (dielectric). Practically ideal dielectric can't exist in nature, so there must be dielectric loss and losses are represented by resistance or conductance

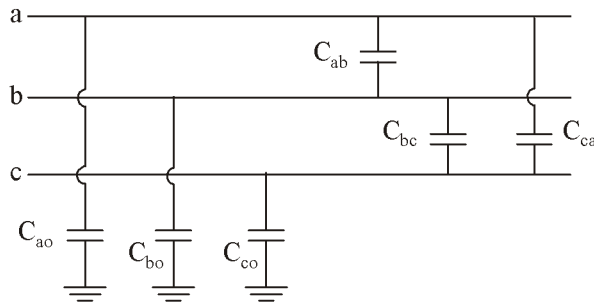


$G$  → Shunt conductance/leakage

So there are four parameters R, L, C, G in power line. In power line transmitted power is represented in "MW" and dielectric loss will be in "Watt". So as compared to power transferred dielectric loss is negligible i.e. leakage is neglected.

All the transmission line contains resistance and inductance and in between the conductors there is capacitance and leakage present through out the transmission line. So that R, L, C, G are called as "distributed parameters".

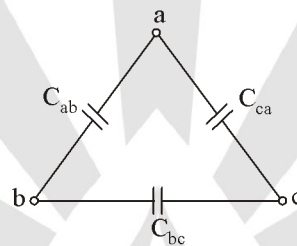
The capacitances in 3 $\phi$  line



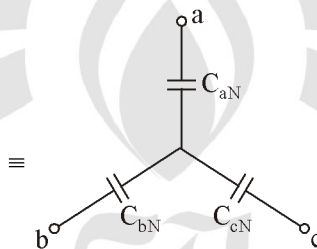
$C_{ao}, C_{bo}, C_{co} \Rightarrow$  Capacitance between line and ground

$C_{ab}, C_{bc}, C_{ca} \Rightarrow$  Capacitance between two lines.

Here capacitance between lines are in delta form and they can be represented as



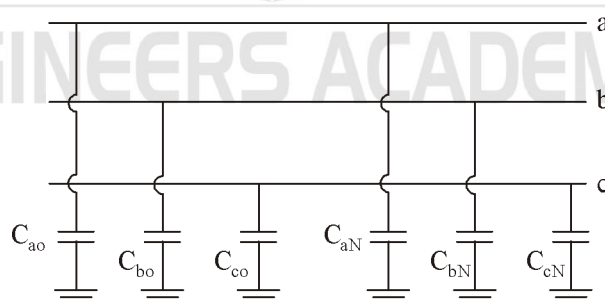
$\Delta$ -Y conversion:



$C_{aN}$  = Capacitance between phase 'a' and Neutral.

$C_{bN}$  = Capacitance between phase 'b' and Neutral.

$C_{cN}$  = Capacitance between phase 'c' and Neutral.

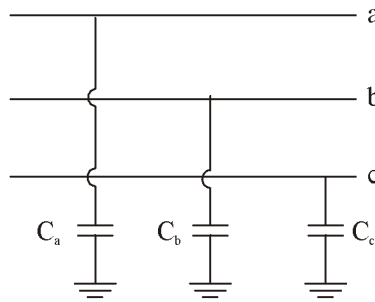


Total capacitance between phase 'a' and ground

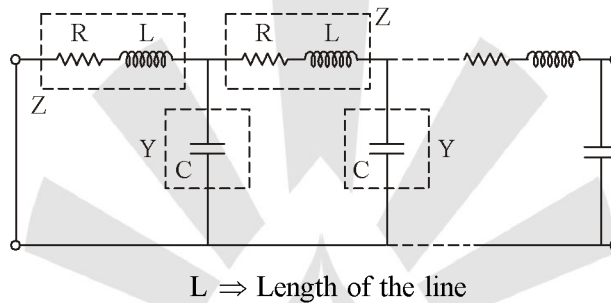
$$C_a = C_{aN} + C_{ao}$$

$$C_b = C_{bN} + C_{bo}$$

$$C_c = C_{cN} + C_{co}$$



## 1.2 DISTRIBUTED PARAMETER MODEL



Per unit length parameters:

Resistance =  $R$

Inductance =  $L$

Capacitance =  $C$

Total series impedance:

$$Z = (R + j\omega L) \times l$$

Total shunt impedance:

$$Z_{sh} = \frac{1}{j\omega Cl} = \frac{1}{j(2\pi f)Cl} = \frac{1}{j(2\pi C)fl}$$

Total shunt admittance:

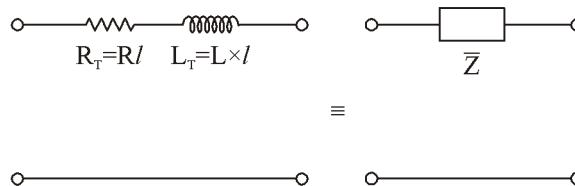
$$Y = j(2\pi C)fl$$

If “ $fl$ ” ( $f \times l$ ) is very small say  $fl < 4000$  Hz km

$\Rightarrow$  Shunt impedance  $Z_{sh} \rightarrow$  very large

$\Rightarrow$  Shunt admittance  $Y \rightarrow$  very small

i.e. effect of shunt capacitance neglected and total series impedance can be represented as concentrated hence the simplified circuit will be



Where  $\bar{Z}$  is the total series impedance

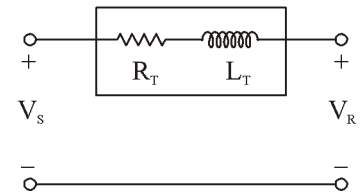
It is said to be short line model

Short line model:

$$(f \times l) < 4000 \text{ Hz km}$$

$f \rightarrow$  Frequency of line

$l \rightarrow$  Length of line



For power line

$$f = 50 \text{ Hz}$$

$\therefore$

$$f \times l < 4000 \text{ Hz km}$$

$\therefore$

$$l < \frac{4000}{50} \text{ km}$$

$$l < 80 \text{ km}$$

$Z_{sh} \rightarrow$  Neglected

### 1.2.1 Medium Line Model

If  $(f \times l)$  is not small say

$$4000 < f \times l < 12000 \text{ Hz km}$$

$$Z_{sh} = \frac{1}{j(2\pi C)(fl)}$$

$f \times l \rightarrow$  Increases

$Z_{sh} \rightarrow$  Decreases

$\therefore Z_{sh}$  is small but it will not be neglected.

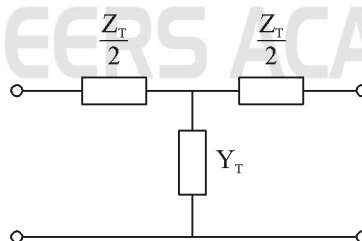
Series impedance

$$Z = (R + j2\pi(fL)) \times l$$

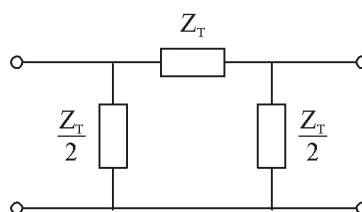
i.e. series impedance will increase.

Shunt admittance is not very large so leakage current can't be neglected but it can be assumed to be concentrated either at the centre of line (i.e. T-model) or equally at the corners of the line (i.e.  $\pi$ -model).

### 1.2.2 Nominal T-Model



Nominal  $\pi$ -Model:



These two are the medium line models.

For power line frequency  $f = 50 \text{ Hz}$

$\therefore 80 \text{ km} < l < 240 \text{ km}$

### 1.2.3 Long Line Model

(Exact distributed if  $fl$  is large i.e. parameter model)

$(f \times l) > 12000 \text{ Hz km}$

For power line frequency

$f = 50 \text{ Hz}$

$l > 240 \text{ km}$

If  $(f \times l)$  is large

$$Z_{sh} = \frac{1}{j(2\pi C)(fl)}$$

$Z_{sh} \rightarrow \text{Decreases}$

Series impedance

$$Z = (R + j\omega L) \times l$$

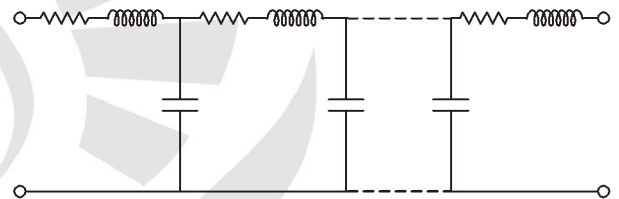
**Note:**

1. Series impedance

$$Z_{(Long \text{ line})} > Z_{(Medium \text{ line})} > Z_{(Short \text{ line})}$$

2. Shunt impedance

$$Z_{sh(Long \text{ line})} > Z_{sh(Medium \text{ line})} > Z_{sh(Short \text{ line})}$$



Exact distributed parameter model i.e. long line

For communication line if  $f = 5 \text{ MHz}$  (In general communication line is used at high frequencies)

Let  $f \times l_1 = 4000 \text{ Hz km}$

$$\Rightarrow l_1 = \frac{4000}{5 \times 10^6} \text{ km}$$

$\therefore l_1 = 80 \text{ cm}$

Similarly  $f \times l_2 = 12000 \text{ Hz km}$

$$\Rightarrow l_2 = \frac{12000}{5 \times 10^6} \text{ km}$$

$\therefore l_2 = 240 \text{ cm}$

If  $f = 5 \text{ MHz}$

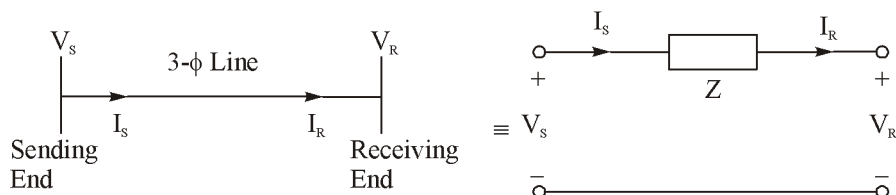
For short line model  $l < 80 \text{ cm}$

For medium line model  $80 \text{ cm} < l < 240 \text{ cm}$

For long line model  $l > 240 \text{ cm}$

So for communication line always long line model is used practically

### 1.2.4 Short Line



Total series impedance

$$\bar{Z} = R + jX_L = R + j\omega L$$

$V_S \rightarrow$  Sending end voltage

$V_R \rightarrow$  Receiving end voltage

$I_S \rightarrow$  Sending end current

$I_R \rightarrow$  Receiving end current

According to the diagram

$$V_S = V_R + Z I_R \quad \dots(i)$$

and

$$I_S = I_R \quad \dots(ii)$$

Transmission Parameter

$$V_S = A V_R + B I_R \quad \dots(iii)$$

$$I_S = C V_R + D I_R \quad \dots(iv)$$

Equation (i) and (ii) also written as

$$V_S = 1 \cdot V_R + Z \cdot I_R$$

$$I_S = 0 \cdot V_R + 1 \cdot I_R$$

When compare with equation (iii) and (iv)

$$A = D = 1$$

$$B = Z$$

$$C = 0$$

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

In case of no load

$$I_R = 0$$

$\therefore$

$$V_S = V_R$$

i.e.

$$V_{R(\text{no load})} = V_S$$

Voltage regulation

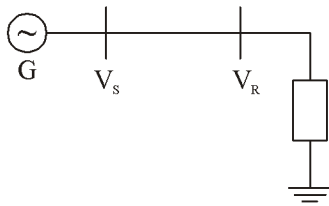
$$V.R. = \frac{V_{R(n)} - V_{R(f)}}{V_{R(f)}} \times 100 = \frac{V_S - V_R}{V_R} \times 100$$

We know that

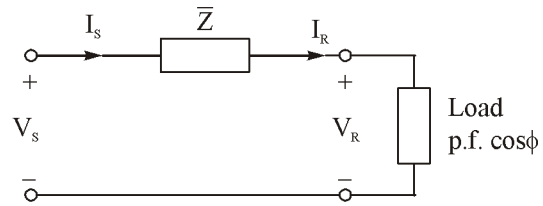
$$\bar{V}_S = \bar{V}_R + Z \bar{I}_R$$

$$\bar{V}_S = \bar{V}_R + \bar{I}_R \times R + j \bar{I}_R \times X \quad \dots(A)$$

When load is connected



(i)



(ii)

At lagging power factor  $\cos\phi$ , receiving end current  $I_R$  lags  $V_R$  by angle  $\phi$ .

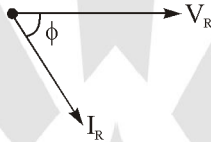
Phasor diagram

$$\bar{V}_s = \bar{V}_R + \bar{I}_R R + j \bar{I}_R X$$

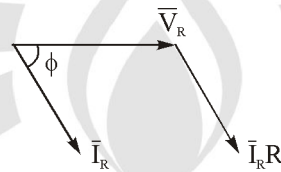
Step-(i) : Draw  $V_R$  along X-axis i.e. reference phase



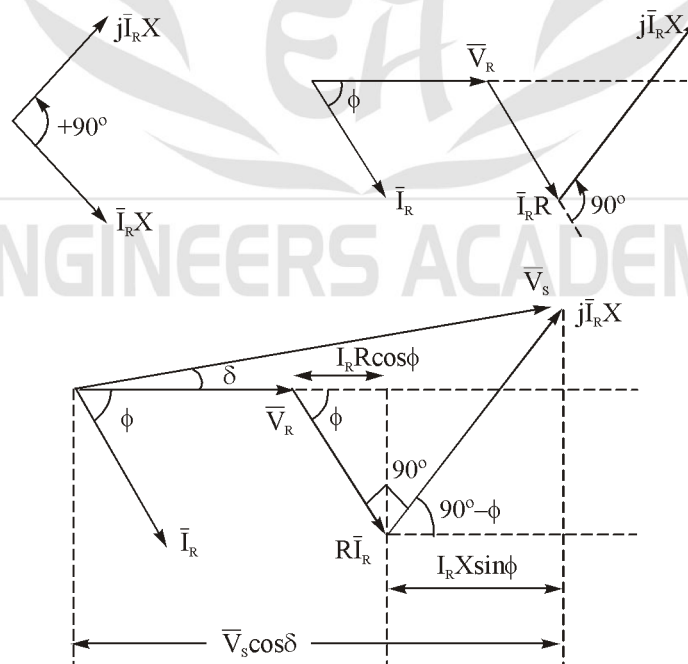
Step-(ii) :  $I_R$  lags  $V_R$  by angle  $\phi$ .



Step (iii) : Add  $\bar{I}_R R$  with  $\bar{V}_R$



Step (iv) : Add  $j \bar{I}_R X_L$  with  $\bar{I}_R R$



As per diagram

$$|\bar{V}_s| \cos\delta = |\bar{V}_R| + |\bar{I}_R| R \cos\phi + |\bar{I}_R| X \sin\phi$$

Due to transient stability criterion the value of ' $\delta$ ' is small, so that  $\cos \delta \approx 1$

$$\therefore |\bar{V}_S| = |\bar{V}_R| + |\bar{I}_R| (R \cos \phi + X \sin \phi)$$

$$\Rightarrow |V_S| - |V_R| = |I_R| (R \cos \phi + X \sin \phi)$$

$$\Rightarrow \frac{|V_S| - |V_R|}{|V_R|} = \frac{|I_R|}{|V_R|} (R \cos \phi + X \sin \phi)$$

i.e. at lagging p.f.  $\cos \phi$

$$\text{Voltage regulation} = \frac{|I_R|}{|V_R|} (R \cos \phi + X \sin \phi)$$

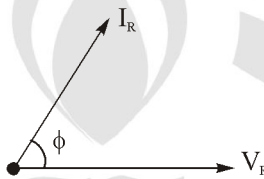
At leading power factor  $\cos \phi$ , i.e.  $I_R$  leads the voltage  $V_R$  by an angle  $\phi$ .

**Phasor Diagram:**

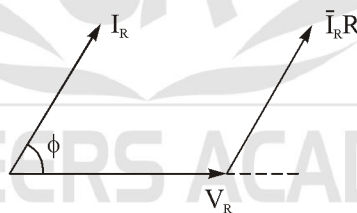
**Step (i) :** Draw  $V_R$  along X-axis



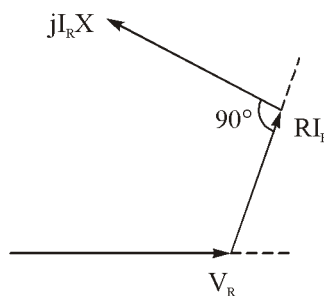
**Step (ii) :**  $I_R$  leads  $V_R$  by an angle  $\phi$ .



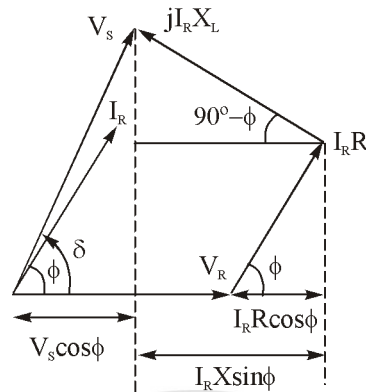
**Step (iii) :** Add  $RI_R$  with  $V_R$



**Step (iv) :** Add  $j I_R X$  with  $RI_R$



Step (v) : Final phasor diagram



As per diagram  $|\bar{V}_s| \cos \delta = |\bar{V}_R| + |\bar{I}_R| R \cos \phi - |\bar{I}_R| X_L \cos \phi$

Due to transient stability criterion the value of  $\delta$  is small. So that  $\cos \phi = 1$ .

$\therefore |\bar{V}_s| = |\bar{V}_R| + |\bar{I}_R| (R \cos \phi - X \sin \phi)$

$\Rightarrow |\bar{V}_s| - |\bar{V}_R| = |\bar{I}_R| (R \cos \phi - X \sin \phi)$

$\Rightarrow \frac{|\bar{V}_s| - |\bar{V}_R|}{|\bar{V}_R|} = \frac{|\bar{I}_R|}{|\bar{V}_R|} (R \cos \phi - X \sin \phi)$

$\therefore \text{Voltage Regulation} = \frac{|\bar{I}_R|}{|\bar{V}_R|} (R \cos \phi - X \sin \phi)$

For maximum voltage regulation

(It is worst regulation)

$$\frac{dVR}{d\phi} = 0$$

At lagging power factor  $\cos \phi$

$\Rightarrow \frac{|\bar{I}_R|}{|\bar{V}_R|} \frac{d}{d\phi} [R \cos \phi + X \sin \phi] = 0$

$\Rightarrow -R \sin \phi + X \cos \phi = 0$

$\Rightarrow X \cos \phi = R \sin \phi$

$\Rightarrow \tan \phi = \frac{X}{R}$

$\therefore \cos \phi = \frac{R}{Z}; \sin \phi = \frac{X}{Z} \quad \left[ \text{Where } Z = \sqrt{R^2 + X^2} \right]$

$$(V.R.)_{\max} = \frac{|\bar{I}_R|}{|\bar{V}_R|} [R \cos \phi - X \sin \phi] = \frac{|\bar{I}_R|}{|\bar{V}_R|} \left[ R \cdot \frac{R}{Z} + X \cdot \frac{X}{Z} \right]$$

$$= \frac{|\bar{I}_R|}{|\bar{V}_R|} \left[ \frac{R^2 + X^2}{Z} \right]$$

$$\therefore (V.R.)_{\max} = \frac{Z |\bar{I}_R|}{|\bar{V}_R|}$$

At lagging power factor  $\tan \phi = \frac{X}{R}$

At leading power factor  $\cos \phi$   $V.R. = \frac{|\bar{I}_R|}{|\bar{V}_R|} [R \cos \phi - X \sin \phi]$

For zero voltage regulation

$$\frac{|\bar{I}_R|}{|\bar{V}_R|} [R \cos \phi - X \sin \phi] = 0$$

$$\Rightarrow \tan \phi = \frac{R}{X}$$

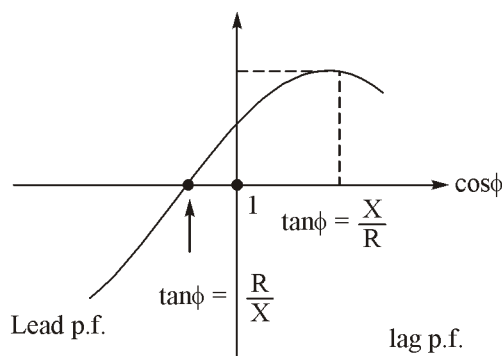
In case of unity power factor

i.e.  $\cos \phi = 1$

i.e.  $\sin \phi = 0$

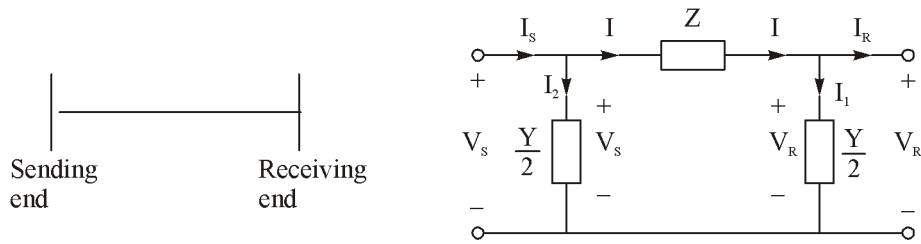
$$\text{Voltage regulation} = \frac{|\bar{I}_R| R}{|\bar{V}_R|}$$

Voltage regulation and power factor curve



## 1.2.5 Medium Line Model

Nominal  $\pi$ -model:



According to the diagram

$$I_1 = \frac{Y}{Z} V_r \text{ and } I = I_r + I_1$$

$$V_s = V_r + IZ$$

$$\Rightarrow V_s = V_r + Z \left[ I_r + \frac{Y}{2} V_r \right]$$

$$\therefore V_s = \left( 1 + \frac{YZ}{2} \right) V_r + Z I_r$$

$$I_2 = \frac{Y}{2} V_s \text{ and } I_s = I + I_2$$

$$I_s = I + \frac{Y}{2} V_s$$

$$\Rightarrow I_s = \left( I_r + \frac{Y}{Z} V_r \right) + \frac{Y}{2} V_s$$

$$\Rightarrow I_s = \left( I_r + \frac{Y}{Z} V_r \right) + \frac{Y}{2} \left[ \left( 1 + \frac{YZ}{2} \right) V_r + Z I_r \right]$$

$$\Rightarrow I_s = Y \left[ 1 + \frac{YZ}{4} \right] V_r + \left[ 1 + \frac{YZ}{2} \right] I_r$$

$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z$$

$$C = Y \left[ 1 + \frac{YZ}{4} \right]$$

Similarly in case of T-model

$$A = D = 1 + \frac{YZ}{2}$$

$$B = Z \left[ 1 + \frac{YZ}{4} \right]$$

$$C = Y$$

**Example 1 :** A 3- $\phi$  100 kV long line has constants per km per conductors as follows: Resistance = 0.5  $\Omega$ , Inductance = 2 mH and capacitance to neutral is 0.015  $\mu$ F. Calculate the voltage required at generating end in order that a load of 10 MVA at 0.8 power factor (lag) may be supplied at 120 kV.

**Solution :**

$$\text{Length} = 100 \text{ km (given) ; } f = 50 \text{ Hz}$$

Total resistance

$$R = 0.5 \times 100 = 50 \Omega$$

Total inductance

$$L = 2 \times 10^{-3} \times 100 = 0.2 \text{ H}$$

Total capacitance

$$C = 0.015 \times 10^{-6} \times 100 = 1.5 \mu\text{F}$$

Total series impedance

$$Z = R + j\omega L = 50 + j \times 2\pi \times 50 \times 0.2 \quad (\omega = 2\pi f)$$

$\therefore$

$$Z = 50 = j62.83$$

$$\bar{Z} = 80.29 \angle 51.5^\circ$$

$$\bar{Y} = j\omega C = j \times 2\pi \times 50 \times 1.5 \times 10^{-6} = j4.71 \times 10^{-4} \text{ S}$$

$$A = D = 1 + \frac{YZ}{2}$$

$$= 1 + \frac{4.71 \times 10^{-4} \angle 90^\circ \times 80.29 \angle 51.5^\circ}{2} = 0.985 \angle 0.68^\circ$$

$$B = Z = 80.29 \angle 51.5^\circ$$

$$C = Y \left[ 1 + \frac{YZ}{4} \right] = 4.71 \times 10^{-4} \angle 90^\circ$$

$$\left[ 1 + \frac{4.71 \times 10^{-4} \angle 90^\circ \times 80.29 \angle 51.5^\circ}{4} \right]$$

$$= 4.68 \times 10^{-4} \angle 90.32^\circ$$

Voltage at receiving end

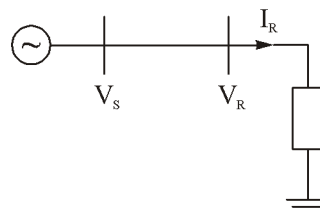
$$V_{R(L-L)} = 120 \text{ kV}$$

Per phase voltage

$$V_{R(ph)} = \frac{120}{\sqrt{3}} = 69.3 \text{ kV}$$

$\therefore$

$$\bar{V}_R = 39.3 \angle 0^\circ \times 10^3 \text{ V}$$



Load

$$S = 10 \text{ MVA}$$

$$\text{p.f. } \cos \phi = 0.8 \text{ lagging} \Rightarrow \phi = 36.9^\circ$$

Load  $S = 3 \times V_{(ph)R} \times I_{(ph)R}$

$\Rightarrow I_R = \frac{10 \times 10^6}{3 \times 69.3 \times 10^3} = 48.1 \text{ A}$

As  $I_R$  lags  $\bar{V}_R$  by  $\phi = 36.9^\circ$

$I_R = 48.1 \angle -36.9^\circ \text{ A}$

Voltage at sending end  $\bar{V}_S = A\bar{V}_R + B\bar{I}_R$

$\Rightarrow \bar{V}_S = 0.985 \angle 0.68^\circ \times 69.3 \times 10^3 \angle 0^\circ + 80.29 \angle 51.5^\circ \times 48.1 \angle -36.9^\circ$

$\Rightarrow \bar{V}_S = 72 \angle 1.4^\circ \text{ kV}$

$V_{S(ph)} = 72 \text{ kV}$

$\therefore V_{S(L-L)} = \sqrt{3} \times V_{S(ph)} = \sqrt{3} \times 72$   
 $= 124.7 \text{ kV}$

To determine:

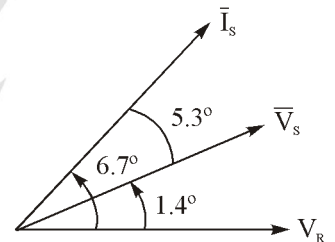
- (i) Sending end power
- (ii) Transmission efficiency
- (iii) Voltage regulation

$\bar{I}_S = C\bar{V}_R + D\bar{I}_R$

$\bar{I}_S = 4.68 \times 10^{-4} \angle 90.32^\circ \times 69.3 \times 10^3 \angle 0^\circ$   
 $+ 0.985 \angle 0.7^\circ \times 48.1 \angle -36.9^\circ$

$\bar{I}_S = 38.254 \angle 6.7^\circ$

$\bar{V}_S = A\bar{V}_R + B\bar{I}_R$



$\bar{I}_S$  leads  $\bar{V}_S$  by  $5.3^\circ$

$\therefore \phi_s = 5.3^\circ$

$\therefore$  Sending end power factor =  $\cos \phi$   
 $= \cos 5.3^\circ$   
 $= 0.99$  (leading)

(i) Sending end power  $P_s = 3 \times V_{s(ph)} \times I_{s(ph)} \times \cos \phi_s = 3 \times 72 \times 10^3 \times 38.23 \times 0.99$   
 $= 8.175 \text{ mW}$

(ii) Receiving end power  $= S_R \cos \phi = 10 \times 0.8 = 8 \text{ MW}$

(iii) Transmission efficiency  $\eta = \frac{P_R}{P_S} = \frac{8}{8.17} = 97.9\%$

(iv) At no load  $I_R = 0$

Voltage  $V_S = AV_R$

$\therefore V_R = \frac{|V_S| \angle \delta}{|A| \angle \alpha}$

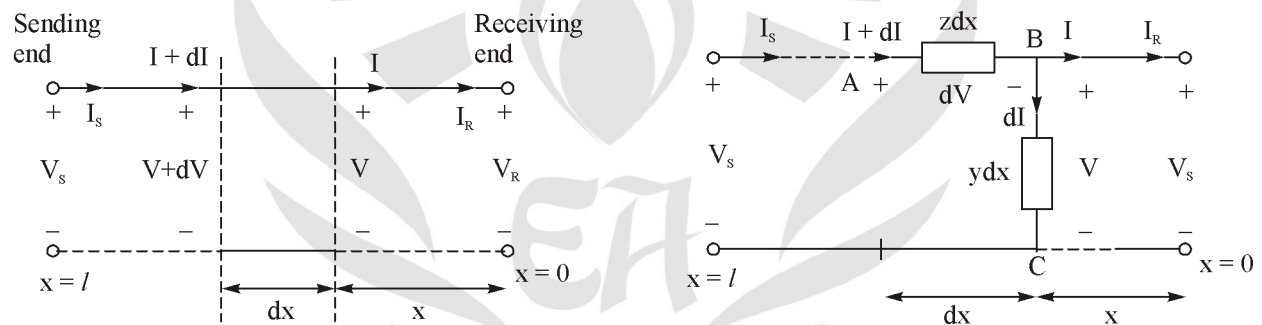
or  $V_{R(NL)} = \frac{72 \angle 1.4^\circ}{0.985 \angle 0.68^\circ} = 73.46 \angle 0.72^\circ$

$$V.R. = \frac{|V_R|_{NL} - |V_R|_{FL}}{|V_R|_{FL}} \times 100 = \frac{73.46 - 69.3}{69.3} \times 100 = 6\%$$

### 1.2.6 Long Line: (Distributed Parameter Model)

Series impedance per unit length  $\bar{z} = r + j\omega L$

Shunt admittance per unit length  $\bar{y} = g + j\omega C$



Let at a distance 'x' from receiving end

voltage = V and current = I

Voltage between point A and B =  $dV$

$$dV = (I + dI)z dx$$

$$dV = z I dx + z dI \cdot dx \quad \dots(i)$$

As  $z dI dx \ll z I dx$

$$dV = z I dx \quad \dots(ii)$$

$$dI = Vy dx$$

$$\Rightarrow \frac{dI}{dx} = Vy \quad \dots(A)$$

$$\frac{dI}{dx} = zI$$

Differentiating w.r.t. x

$$\Rightarrow \frac{d}{dx} \left[ \frac{dV}{dx} \right] = z \left[ \frac{dI}{dx} \right]$$

Now from equation (A)

$$\Rightarrow \frac{d^2V}{dx^2} = yzV$$

$$\Rightarrow \frac{d^2V}{dx^2} - yzV = 0$$

Let  $yz = \gamma^2$  (Constant)

$$\Rightarrow \frac{d^2V}{dx^2} - \gamma^2V = 0$$

After solving the equation  $V = C_1e^{\gamma x} + C_2e^{-\gamma x}$  ...(B)

Where  $C_1, C_2$  are constants.

Now,  $I = \frac{1}{z} \cdot \frac{dV}{dx}$

$$\Rightarrow I = \frac{1}{z} \cdot \frac{d}{dx} [C_1e^{\gamma x} + C_2e^{-\gamma x}]$$

$$\Rightarrow I = \frac{1}{z} \cdot \gamma [C_1e^{\gamma x} - C_2e^{-\gamma x}]$$

Let  $\frac{\gamma}{z} = \frac{\sqrt{yz}}{z} = \sqrt{\frac{y}{z}} = \frac{1}{Z_0}$

$$\therefore \sqrt{\frac{y}{z}} = \frac{1}{Z_0}$$

$$\Rightarrow I = \frac{1}{Z_0} [C_1e^{\gamma x} - C_2e^{-\gamma x}]$$

At receiving end  $x = 0$   
 $V = V_R$  and  $I = I_R$

$$\therefore V_R = C_1 + C_2 \quad \text{...(iii)}$$

$$I_R = \frac{1}{Z_0} [C_1 - C_2] \quad \text{...(iv)}$$

From equation (iii) and (iv)  $C_1 + C_2 = V_R$

$$C_1 - C_2 = Z_0 I_R$$

$$C_1 = \frac{1}{2} [V_R + Z_0 I_R]$$