STRENGTH OF MATERIALS

CHAPTER 1

PROPERTIES OF METALS, STRESS AND STRAIN

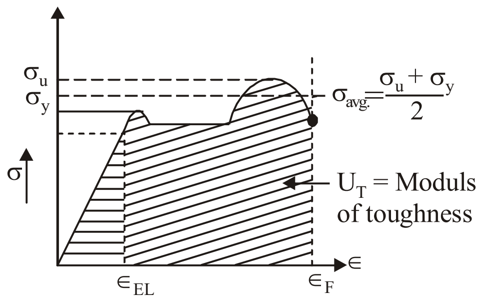
IMPORTANT MECHANICAL PROPERTIES

- ***Elasticity :***
- It is the property by virtue of which a material deformed under the load is **enabled** to return to its original dimension when the load is removed.
- If body regains **completely** its original shape then it is called **perfectly** elastic body.
- **Elastic limit** marks the **partial** break down of elasticity beyond which removal of load result in a degree of **permanent deformation.**
- Steel, Aluminium, Copper, may be considered to be perfectly elastic **within certain limit.**
- ***Plasticity :***

  The characteristics of the material by which it undergoes **inelastic strain** beyond those at the **elastic limit** is known as plasticity.
- This property is particularly useful in operation of **pressing** and **forging.**

  When large deformation occurs in a **ductile** material loaded in **plastic** region, the material is said to undergo **plastic flow.**
- ***Ductility :***
- It is the property which permits a material to be drawn out **longitudinally** to a reduced section, under the action of **tensile force.**
- A ductile material must posses a high degree of plasticity and strength. Ductile material must have **low** degree of elasticity.
- This is useful in **wire drawing.**
- ***Brittleness :***
- It is lack of ductility. Brittleness implies that it can **not** be drawn out by tension to smaller section
- In brittle material failure take place under load **without** significant deformation. Ordinary **Glass** is nearly **ideal** brittle material.
- Cast iron, **concrete** and ceramic material are brittle material.

- ***Malleability***
- It is the property of a material which permits the material to be **extended** in **all direction** without rupture.
- A malleable material posses a **high degree** of plasticity, but **not** necessarily **great strength.**
- ***Toughness***
- It is the property of material which enables it to absorb energy **without fracture.**
- Modulus of toughness $U_T = \left(\dfrac{\sigma_u + \sigma_y}{2}\right)\in_f$
- It is desirable in material which is subjected to **cyclic** or **shock loading.**
- It is represented by area under **stress-strain** curve for material upto fracture.
- **Bend test** used for common comparative test for toughness.



- ***Hardness***
- It is the ability of a material to resist **indentation** or **surface abrasion.**
- Brinell hardness test is used to check hardness.

$$\text{Brinell hardness number} = \frac{P}{\frac{\pi D}{2}\left[D - \sqrt{D^2 - d^2}\right]}$$

Here, P = Standard load
D = Diameter of steel ball (mm)
d = Diameter of indent (mm)

- ***Strength***
- This property enables material to resist fracture under load.
- This is most important property from **design** point of view.
- Load required to cause fracture, divided by area of test specimen, is termed as **ultimate strength.**

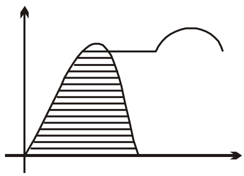
- ***Creep***
- Creep is a permanent deformation which is recorded with passage of time at constant loading. It is plastic deformation (permanent and non recoverable) in nature.

Note : The temperature at which creep is uncontrollable is called **Homologous Temperature.**

- ***Fatigue***
- Due to cyclic or reverse cyclic loading fracture failure may occur if total accumulated strain energy exceeds the toughness. Fatigue causes rough fracture surface even in ductile metals.
- ***Resilience***
- It is total elastic strain energy. Which can be stored in the given volume of metal and can be released after unloading.
- ***Modules of Resilience***
- Higher toughness is desirable property in materials used for gears, chains, where higher resilience is desired in spring.

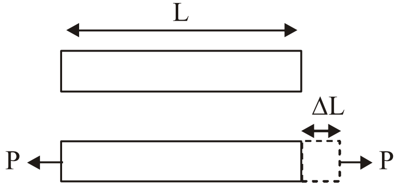
STRESS AND STRAIN

- ***STRESS ($\frac{\sigma_y^2}{2E}$ N/m²)***



- It is the resistance offered by the body to deformation
  - Nominal stress (Engineering stress) = $\frac{\text{Load}}{\text{Original Area}}$
  - Actual/True stress = $\frac{\text{Load}}{\text{Changed (Actual) Area}}$
- ***STRAIN***
- Deformation per unit length in the direction of deformation is known as strain.

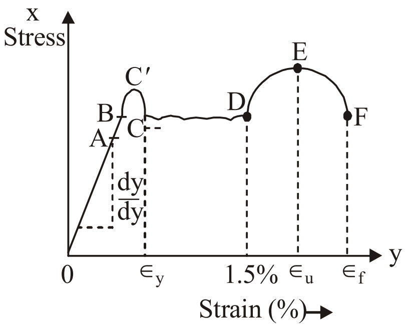
Strain = $\frac{\Delta L}{L}$;



- It is a **dimensionless** quantity. Strain is more basic then stress.

ENGINEERING STRESS-STRAIN CURVE OF MILD STEEL FOR TENSION UNDER STATIC-LOADING

OA— Straight line (proportional region, Hooke***'s law is valid)***



OB — Elastic region
BC — Elasto plastic region
CD — Perfectly plastic region
DE — Strain hardening
EF — Necking region
A — Limit of proportionality
B — Elastic limit
C — Lower yield point
F — Fracture point
C' — Upper yield point
D — Strain hardening starts

E—Ultimate point or maximum stress point

- ***Limit of Proportionality***
- It is the stress at which the stress-strain curve **ceases** to be a straight line.
- **Hooke's law is valid upto proportional limit.**
- ***Elastic Limit***
- It is the point on the stress-strain curve upto which the materials remains elastic.
- Upto this point there is **no permanent** deformation after removal of load.
- ***Plastic Range***
- It is the region of the stress-strain curve between the elastic limit and point of rupture.
- ***Yield Point***
- This point is just beyond the elastic limit, at which the specimen undergoes an appreciable increase in length **without** further increase in the load.
- ***Rupture Strength***
- It is the stress corresponding to the failure point 'F' of the stress-strain curve.
- ***Proof Stress***
- It is the stress necessary to cause a **permanent extension** equal to defined percentage of gauge length.
- Generally It is designed for the material which don't have any clear yield. Point.

➲ ***(Young's Modulus).***

❖ It is constant of proportionality which is defined as the intensity of stress that causes unit strain.

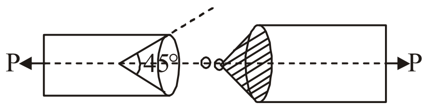
❖ Plastic strain is 10 to 15 times elastic strain.

❖ Fracture strain ($\in_f$) depends on **percentage carbon** in steel.

❖ When carbon percentage increases then fracture strain decreases and yield stress increases.

TYPES OF TENSION FAILURE IN METAL

- ***Ductile metal (Shear failure) :*** Ductile materials are weak in shear.

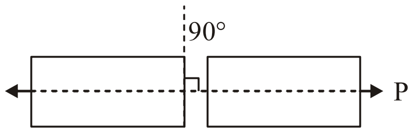


Failure plane is at **45°**

Cup-cone fracture

Shear strength < Tensile strength ≤ Compressive Strength

- ***Brittle metal :*** These are weak in tension.



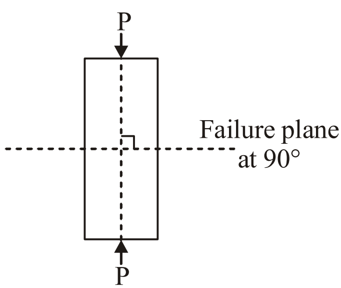
✪ Failure plane at **90°** with longitudinal direction

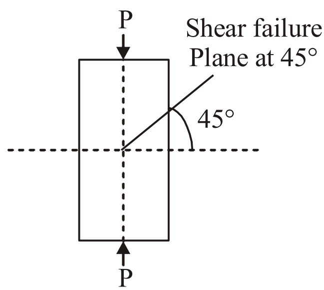
✪ Necking is not formed and failure is due to tension failure.

✪ Tensile strength < Shear strength < Compressive strength

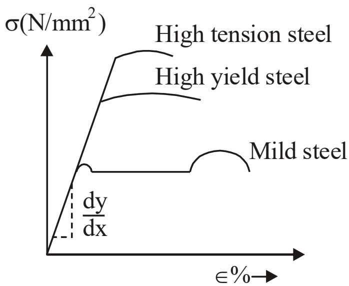
TYPE OF FAILURE IN COMPRESSION

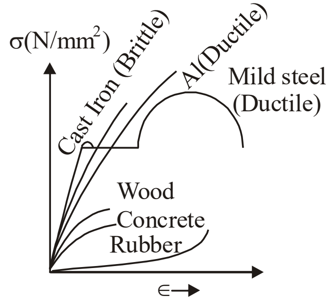
Ductile material **Brittle material**





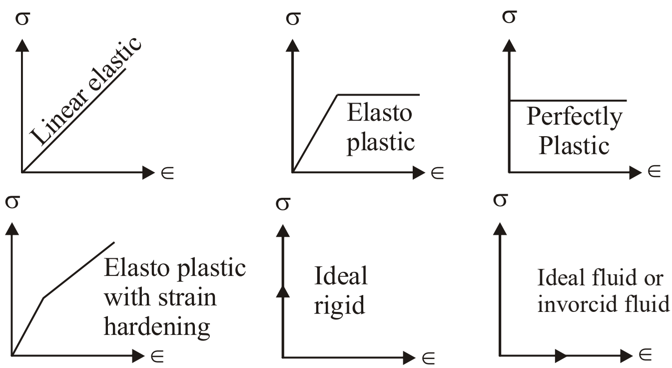
STRESS-STRAIN DIAGRAM FOR VARIOUS TYPE OF STEEL/ MATERIAL





All grades of steel have same young's modulus but different yield stress.

- ***DUCTILE MATERIAL***
- If post elastic strain is greater than 5%, it is called ductile material.
- It undergoes large permanent strains before failure.
- Large reduction in area before fracture. e.g. **lead,** mild steel, copper
- ***BRITTLE MATERIAL***
- If post elastic strain is less than 5%. It is called brittle material.
- It fails with only little elongation after the proportional limit is exceeded.
- Very less reduction in area before fracture, e.g. Bronze, **Rubber,** Glass
- ***Behaviour of Various Material***



Where σ = Stress, ∈ = Strain

- **'Mild steel'** is **more** elastic than **'Rubber'.**

- ***HOOKE'S LAW :*** Valid upto proportional limit.
- When a material behaves elastically and exhibits a linear relationship between stress and strain, it is called linearly elastic. For such materials stress (σ) is directly proportional to strain ($\in$).

$$\boxed{\sigma \propto \in} \rightarrow \boxed{\sigma = E.\in}$$

Here, σ = Stress

$\in$ = Strain

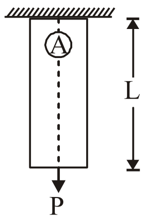
E = Young modulus of elasticity

- $E_{\text{cast iron}} \approx \frac{1}{2} E_{\text{steel}}$
- $E_{\text{Aluminium}} \approx \frac{1}{3} E_{\text{steel}}$

Where 'E' is young modulus.

- ***AXIAL ELONGATION (Δ) OF PRISMATIC BAR DUE TO EXTERNAL LOAD***

$$\Delta = \frac{PL}{AE}$$



Here, P = Load applied

L = Length of bar

A = Area of bar

E = Young modulus of elasticity.

$$\Delta = \frac{P}{\frac{EA}{L}} = \frac{P}{K}$$

Here, K = AE/L = Axial stiffness of bar

AE= Axial rigidity

EI/L= Flexural stiffness

EI = Flexural rigidity

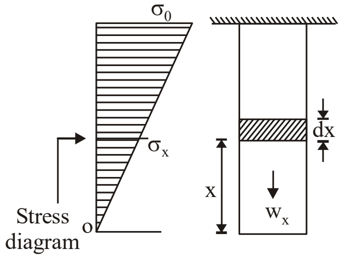
- For nonuniform bass :

$$\boxed{\frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} - \frac{L_n}{A_n}\right]}$$

- L_1, L_2, L_3 : Length of various section.
- A_1, A_2 Area of corresponding section.

DEFLECTION OF BAR (Δ) DUE TO SELF-WEIGHT

➲ ***Prismatic bar***



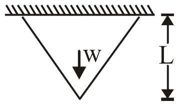
$$\Delta = \frac{WL}{2AE} = \frac{\gamma L^2}{2E}$$

γ = specific weight or weight density.

Here, W = Self weight

Conical bar

$$\Delta = \frac{\gamma L^2}{6E} = \frac{1}{3} \times \text{ Deflection of Prismatic bar of same length}$$

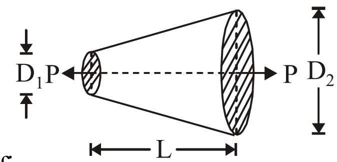


Here, γ = Specific weight
L = Length of bar
E = Young's modulus

DEFLECTION (Δ) OF TAPERED BAR

➲ ***Circular tapering bar***

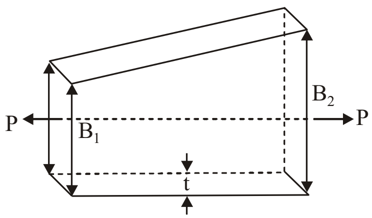
$$\Delta = \frac{4PL}{\pi E D_1 D_2}$$



Here, P = Load applied
L = Length of bar

D_1 and D_2 are diameter as shown in fig.

➲ ***Rectangular tapering bar***

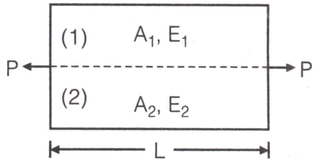


$$\Delta = \frac{PL \log_e\left(\frac{B_2}{B_1}\right)}{E.t(B_2 - B_1)}$$

Here, t = thickness
P = Load applied
E = Young modulus

EQUIVALENT YOUNG'S MODULUS OF PARALLEL COMPOSITE BAR

$$\text{Equivalent} = \frac{A_1E_1 + A_2E_2}{A_1 + A_2}$$



Here, P = Load

A_1 = Area of first bar

A_2 = Area of second bar

E_1 = Young's modulus of first bar

E_2 = Young's modulus of second bar

L = Length of bar

ELASTIC CONSTANTS:

- Elastic constants are those factor whose determine the deformation produced by a given stress system acting on material.

$$\text{Modulus of elasticity (E)} = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$$

$$\text{Modulus of rigidity (G)} = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$\text{Bulk modulus (K)} = \frac{\text{Direct stress}}{\text{Vlumetric strain}} = -\left(\frac{\Delta D}{\frac{\Delta V}{V}}\right)$$

POISSON'S RATION (μ)

$$\mu = \frac{-(\text{Lateral strain})}{\text{Longitudinal strain}} \; ; \; \mu = \left|\frac{\text{Lateral strain}}{\text{Longitudinal strain}}\right|$$

Under uniaxial loading

$0 < \mu < 0.5$

$\mu = 0$ for cork

$\mu = 0.5$ For perfectly plastic body **(Rubber)**

$\mu = 0.25$ to 0.42 for elastic metals

$\mu = 0.1$ to 0.2 for concrete

$\mu = 0.286$ mild steel

μ is greater for ductile metals than for brittle metals.

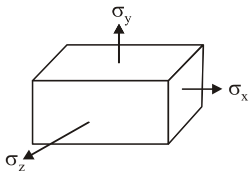
VOLUMETRIC STRAIN UNDER TRI-AXIAL LOADING

Here, σ_x = Stress in x-direction

σ_y = Stress in y-direction

σ_z = Stress in z-direction

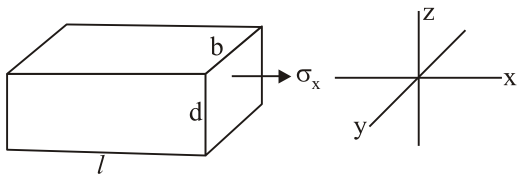
$\in_v$ = Volumetric strain



$$\in_v = \in_x + \in_y + \in_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1-2\mu)$$

Under hydrostatic loading, $\sigma_x = \sigma_y = \sigma_z = \sigma$, $\therefore \in_V = \frac{3\sigma}{E}(1-2\mu)$

UNI-AXIAL LOADING ON RECTANGULAR PARALLELEPIPED

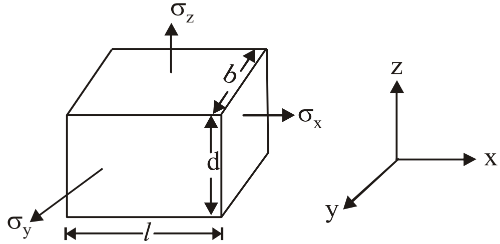


$$\in_x = \frac{\Delta l}{l} = \frac{\sigma_X}{E}\ ;\ \in_y = \frac{\Delta b}{b} = -\frac{\mu\sigma_X}{E}\ ;\ \in_z = \frac{\Delta d}{d} = -\frac{\mu\sigma_X}{E}$$

✪ Here, $\in_x, \in_y$ and $\in_z$ are strain in x,y and z directions respectively.

Δl, Δb and Δd are change in length, width and depth respectively. l,b and d are original length, width and depth respectively.

TRIAXIAL LOADING ON RECTANGULAR PARALLELEPIPED

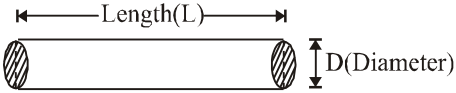


$$\in_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta l}{l}\ ;\ \in_y = \frac{\sigma_y}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_z}{E} = \frac{\delta b}{b}$$

$$\in_z = \frac{\sigma_z}{E} - \frac{\mu\sigma_x}{E} - \frac{\mu\sigma_y}{E} = \frac{\delta d}{d}$$

✪ ***Sign convention:*** Tensile is positive, and compressive is negative.

VOLUMETRIC STRAIN OF CYLINDRICAL BAR



$\in_v$ = Longitudinal strain + (2 × Diametric strain)

VOLUMETRIC STRAIN OF SPHERE

$\in_V$ = 3 × Diametric strain

MATRIX REPRESENTATION OF STRESS AND STRAIN

3-D stress matrix $\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$

3-D strain matrix $\begin{bmatrix} \in_{xx} & \dfrac{\phi_{xy}}{2} & \dfrac{\phi_{xz}}{2} \\ \dfrac{\phi_{yx}}{2} & \in_{yy} & \dfrac{\phi_{yz}}{2} \\ \dfrac{\phi_{zx}}{2} & \dfrac{\phi_{zy}}{2} & \in_{zz} \end{bmatrix}$

RELATION BETWEEN E, G, K, μ

- $E = 3K(1 - 2\mu)$
- $E = 2G(1 + \mu)$
- $E = \dfrac{9KG}{3K + G}$
- $\mu = \dfrac{3K - 2G}{6K + 2G}$

Here, E = Young's modulus, G = shear modulus

K = Bulk modulus, μ = Poisson ratio

Material	Number of Independent elastic constant
Homogeneous & Isotropic	2
Orthotropic (Wood)	9
Anisotropic	21

STRAIN ENERGY

It is the ability of material to absorb energy when it is strained

$$U = \frac{1}{2} P \times \delta = \frac{1}{2} T \times \theta$$

Here,
- P = Applied load
- δ = Elongation due to applied load
- T = Applied torque
- θ = Angle of twist due to applied torque

- ***Resilience:*** Ability of a material to absorb energy in the **elastic region** when it is strained.

  $$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2} P \times \delta$$

- ***Proof Resilience: Maximum*** energy absorbing capacity of a material in the **elastic region** is called proof resilience.

  $$= \text{Area under P-}\delta\text{ curve} = \frac{1}{2} P_{EL} \times \delta_{EL}$$

Here
- P_{EL} = Load at elastic limit
- δ_{EL} = Modulus of elasticity

THERMAL STRESS AND STRAIN

$\sigma_{Th,stress} = E\,\alpha\,T; \quad \Delta = L\,\alpha\,T$

$$\text{Strain} = \frac{L\alpha T}{L} = \alpha T\ ;\ \sigma_{steel} = \alpha_{castiron} = 12 \times 10^{-6} / {}^\circ C$$

$$\alpha_{Aluminimum} > \alpha_{Brass} > \alpha_{Copper} > \alpha_{Steel}$$

Here,
- σ = Thermal stress
- α = Coefficient of thermal expansion in/^{o}C
- T = Temperature change
- Δ = Change in length

- When bar is free to expand than there will be no thermal stress due to change in temperature.

SHEARS FORCE AND BENDING MOMENT

➲ ***Simply Supported Beam***

✪ If the ends of a beam are made to rest freely on supports it is called a simply (freely) supported beam.

➲ ***Fixed Beam***

✪ If a beam is fixed at both ends it is called fixed beam its another name is encastre or built-in beam.

➲ ***Cantilever Beam***

✪ If a beam is fixed at one end while other end is free, it is called cantilever beam.

➲ ***Continuous Beam***

✪ If more than two supports are provided to beam, it is called continuous beam.

SHEAR FORCE

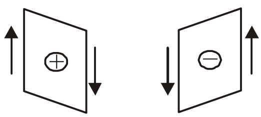
✪ It is the internal resistance developed at any section to maintain free body equilibrium of either left or right part of the section.

✪ It may be horizontal or vertical.

✪ Shear force at any section is algebraic sum of all transverse forces either from left or right of that section.

➲ ***Sign Convention***

❖ Shear force having an upward direction to the left hand side of section or downward direction to the right hand side of section will be taken positive and vice-versa.



BENDING MOMENT

✪ Bending moment at any section is the internal reaction due to all the transverse force either from left side or from right side of that section.

✪ It is equal to algebraic sum of moments at that section either from left or from right side of that section.

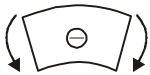
✪ Bending moment is different from twisting moment.

➲ ***Sign convention of Bending moment***

❖ A bending moment causing concavity upward will be taken as positive and called sagging bending moment.

Sagging

❖ A bending moment causing convexity upward will be taken as negative and will be called a hogging bending moment.



Hogging

RELATIONSHIP BETWEEN BENDING MOMENT (M), SHEAR FORCE (S) AND LOADING RATE (w)

✪ Rate of change of shear force is equal to load

$\frac{dS}{dX} = w$ Here, w = Load per unit length

❖ Negative slope represent downward loading.

✪ Rate of change of bending moment along the length beam is equal to shear force.

$$-\frac{dM}{dX} = S_x$$

❖ At hinge, bending moment will be zero.

✪ Bending moment is maximum or minimum when shear force is zero or changes sign at a section.

If degree of shear force curve = n then

degree of shear force curve = n + 1

and degree of bending moment curve = n + 2

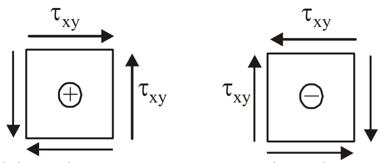
✪ Point of contra-fixture/inflection point is a point where bending moment changes its sign.

✪ Principal stress are maximum or minimum normal stress which may be developed on a body, when a body is subjected to any normal stress.

❖ The plane of principal stress carry zero shear stress.

SIGN CONVENTIONS

- Tensile stress is considered positive and compressive stress is negative.



Positive shear stress   Negative shear stress

- Angle 'θ' is considered positive if it is in anti-clockwise direction.
- Shear stress acting on a positive face of an element is considered positive if it is acts in positive direction of one of the coordinate axes and negative if acts in the negative direction of the axes. Similarly on a negative force of an element is positive if it act in negative direction oft he axes and negative if it act in the positive direction.
- Normally the reference plane taken are major principal plane or vertical plane.

ANALYTICAL METHOD OF ANALYSIS

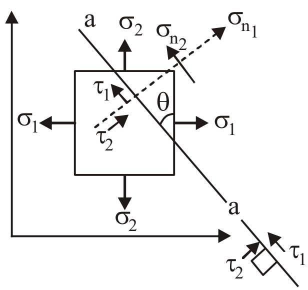
- If σ_1 and σ_2 are given principal stress as shown in figure, then normal and shear stress on plane a-a which is inclined at angle 'θ' from major principal plane ($\sigma_1 > \sigma_2$)

$$\therefore \ \sigma_{n_1} = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$

$$\sigma_{n_1} = \frac{\sigma_1+\sigma_2}{2} + \left(\frac{\sigma_1-\sigma_2}{2}\right).\cos 2\theta$$

$$\sigma_{n_2} = \frac{\sigma_1+\sigma_2}{2} - \left(\frac{\sigma_1-\sigma_2}{2}\right).\cos 2\theta$$

$$\tau_1 = \tau_2 = -\left(\frac{\sigma_1-\sigma_2}{2}\right)\sin 2\theta$$



- $\sigma_{n_1} + \sigma_{n_2} = \sigma_1 + \sigma_2 = \text{constant}$

If $\theta = 45°$ or $135°$ then, $\tau_1 = \tau_2 = \tau_{max} = -\left(\frac{\sigma_1-\sigma_2}{2}\right)$

On the Plane of τ_{max}.

$$\sigma_{n_1} = \sigma_{n_2} = \frac{\sigma_1+\sigma_2}{2}$$